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Effects of Medium Inhomogenieties on Surface-Generated Ambient Noise

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ABSTRACT

The effects on surface-generated ambient noise of medium inhomogenieties caused by small sound-speed perturbation are considered. A noise-generation model combined with the wave propagation solution in a random medium is applied to a typical oceanic environment to study the characteristics of the surface-generated noise field. Based upon leading-order analysis, the effects of medium inhomogenieties on the noise field, including the wavenumber spectrum, noise intensity, and spatial correlation, are analyzed. The results show that the sound-speed perturbations have an effect equivalent to that of medium absorption so that the efficiency of waveguide propagation is degraded. In particular, the normal modes become less prominent, which in turn reduces the noise intensity in the waveguide. Furthermore, the spatial coherence of the noise field decreases with increasing randomness in the medium, indicating that the coherence of the noise field is partly attributable to the characteristics of the medium inhomogenieties.

Key Words: ambient noise, medium inhomogenieties, wavenumber spectrum, noise intensity, spatial correlation

I. Introduction

ACOUSTIC WAVE PROPAGATION in an ocean waveguide is subject to loss of energy due to various factors. These include intrinsic absorption of seawater, geometric spreading of propagation, boundary transmission and scattering, and volume scattering; each attenuation mechanism is induced by specific physical processes. Volume scattering may be caused by interaction between acoustic wave and objects obstructing its propagation, or by random perturbations of the acoustic properties, including the density and sound speed, of the medium itself. In seawater, perturbations of the density and sound speed may originate from oceanographic mixing and internal waves, and it has been reported that the variation of the sound speed is an order of magnitude larger than that of the density (Chernov, 1960). As a result, a substantial amount of research has focused on scattering due to sound-speed variation (Dozier and Tappert, 1977; Desaubies, 1978; Essen *et al.*, 1983). The subject of this paper is the effects of sound-speed perturbations in the medium on ambient noise generated by surface random sources.

Our interest in surface-generated ambient noise in an oceanic environment stems from the fact that it is an important part of underwater sound. Due to its persistent existence and wide coverage in the noise spectrum, surface-generated ambient noise is likely to contaminate any signal propagating in an ocean. In view of this, it has been regarded as a destructive factor in traditional analysis, particularly in sonar applications. However, surface-generated ambient noise also reflects many useful properties of the marine environment, including the surface itself and the media supporting wave propagation, thus allowing us to extract the desired information from ambient noise. In either case, to implement practical applications, it is necessary that the composition of ambient noise and any physical processes affecting it be extensively explored and well understood.

The problem under consideration is schematically shown in Fig. 1, which shows a water column with density ρ_1 and sound speed c_1 overlying a semi-infinite fluid half-space with density ρ_2 and sound speed c_2, where the sound speed in each medium is subject to small random perturbations. Near the top of the water column, there exists an infinite plane of monopoles, used to simulate the noise sources between the air and water interface. This noise generation model was first proposed by Kuperman and Ingenito (1980), and later was applied by Schmidt and Kuperman (1988) to study the effects of seismic waves on the estimation of low-frequency ambient noise level. The present analysis considers the effects of medium inhomogenieties due to sound-speed variations on the characteristics of the noise field, thus supplying further information about the noise field subject to the influence of environmental variability.

The analysis requires the use of formulations for wave propagation in a random medium as well as for the cross-correlation function of the noise field. In the following sections, we shall first formulate the problem and then give numerical examples to demonstrate various effects of medium randomness on the noise field.

II. Formulations

In this section, we will first summarize the formulation for wave propagation in a random medium, and then derive the cross-correlation function of the noise field. The derivation process is parallel to those employed byKuperman and Ingenito (1980), Liu *et al.* (1993), and Tang and Frisk (1991) for noise generation, scattering, and propagation in a random medium, respectively.

1. Wave Propagation in a Random Medium

Consider an acoustic monotonic wave with time dependent $e^{-i\omega t}$ propagating in a medium i whose sound speed is subject to a small random perturbation:

$$c_i = \langle c_i \rangle + c_i' , \tag{1}$$

where $\langle c_i \rangle$ is the ensemble average of the sound speed and $c_i'(x,y,z)$, with the property $\langle c_i' \rangle = 0$, is the random perturbation; with the time-dependent term suppressed, the acoustic wave satisfies the Helmholtz equation

$$\nabla^2 p_i + k_i^2 p_i = 0 , \tag{2}$$

where $k_i(x,y,z) = \omega/c_i(x,y,z)$ is the wavenumber. Under the assumption of small perturbation of the sound speed, i.e., $|c_i'| << |\langle c_i \rangle|$, it is readily seen that the wavenumber can be replaced by the following approximation:

$$k_i^2 = \left(\frac{\omega}{\langle c_i \rangle + c_i'} \right)^2 \cong \left[\frac{\omega}{\langle c_i \rangle} \left(1 - \frac{c_i'}{\langle c_i \rangle} \right) \right]^2$$

$$\cong \langle k_i \rangle^2 \left(1 - \frac{2c_i'}{\langle c_i \rangle} \right) . \tag{3}$$

According to small perturbation analysis, the total acoustic field p_i may be decomposed into a coherent mean field $\langle p_i \rangle$ and an incoherent scattered field p_i^s:

$$p_i = \langle p_i \rangle + p_i^s , \tag{4}$$

where $|p_i^s|$ is assumed to be of the same order as $|c_i'|$ so that $|p_i^s| << |\langle p_i \rangle|$. Substituting Eqs. (3) and (4) into Eq. (2), taking the ensemble average, and then subtracting the resulting equation from Eq. (2), one can obtain the governing equations

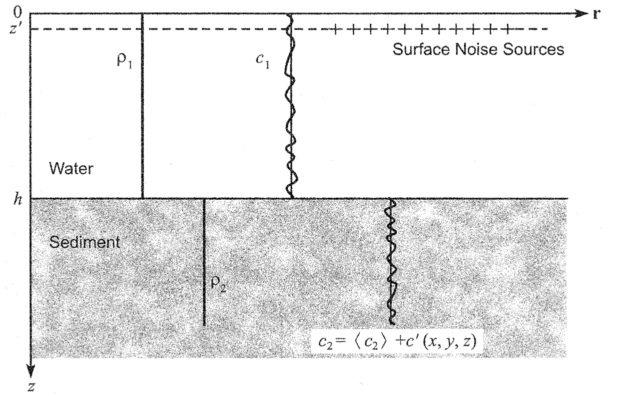

Fig. 1. Environmental model: Noise field generated by surface random sources in an oceanic environment with perturbed sound-speed distributions.

for the mean field and the scattered field (Tang and Frisk, 1991):

$$\nabla^2 \langle p_i \rangle + \langle k_i \rangle^2 \langle p_i \rangle = - \langle k_i \rangle^2 \langle \varepsilon_i p_i^s \rangle , \tag{5}$$

$$\nabla^2 p_i^s + \langle k_i \rangle^2 p_i^s = - \varepsilon_i \langle k_i \rangle^2 \langle p_i \rangle + \langle k_i \rangle^2 \left(\langle \varepsilon_i p_i^s \rangle - \varepsilon_i p_i^s \right) , \tag{6}$$

where $\varepsilon_i = -2c_i'/\langle c_i \rangle$. It is noted that the term on the right-hand-side of Eq. (5) is built in to represent the effect of randomness on the mean field. Dropping the second-order terms in Eq. (6) and invoking the Green's theorem, we have

$$p_i^s(\boldsymbol{R}) = \frac{\langle k_i \rangle^2}{4\pi} \int_{V'} \varepsilon_i(\boldsymbol{R}') \langle p_i(\boldsymbol{R}') \rangle G_i(\boldsymbol{R}; \boldsymbol{R}') dV' , \tag{7}$$

where $\boldsymbol{R}=(\boldsymbol{r},z)$ and $G_i(\boldsymbol{R}; \boldsymbol{R}')$ is the Green's function, representing the acoustic pressure at point $\boldsymbol{R}$ due to the unit point source at $\boldsymbol{R}'$ in the volume $\boldsymbol{V}'$. Substituting Eq. (7) into Eq. (5) results in an integro-differential equation which must be satisfied by the mean field:

$$\nabla^2 \langle p_i(\boldsymbol{R}) \rangle + \langle k_i \rangle^2 \langle p_i(\boldsymbol{R}) \rangle$$

$$= - \frac{\langle k_i \rangle^4}{4\pi} \int_{V'} \langle \varepsilon_i(\boldsymbol{R}) \varepsilon_i(\boldsymbol{R}') \rangle \times \langle p_i(\boldsymbol{R}') \rangle G_i(\boldsymbol{R}; \boldsymbol{R}') dV' . \tag{8}$$

The solution of Eq. (8) depends upon the randomness of the medium and is, in general, not available in closed form so that a numerical procedure must be invoked. However, under certain circumstances, a semi-analytical form expressible in terms of amenable integrals can be obtained. For example, the solution for the random half-space with a special class of sound-speed perturbation was derived by Frisk (1979) and Tang and Frisk (1991), and shall be applied in a later section in this study.

2. Cross-Correlation Function of the Noise Field

The objective of this derivation is to obtain the cross-correlation function of the noise field resulting from random noise sources. Applying the concept of plane-wave decomposition of the noise field (Jensen *et al.*, 1994), the pressure field in a layer i due to a point source of strength S_ω at $(\boldsymbol{r}_s, z_s)$ can be represented by the Fourier integral:

$$p_i^{ps}(\boldsymbol{r}, z; \boldsymbol{r}_s, z_s) = \frac{1}{2\pi}\int d^2\boldsymbol{k} S_\omega(\boldsymbol{r}_s, z_s)\tilde{p}_i(\boldsymbol{k}, z)e^{-i\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}_s)}, \quad \mathbf{(9)}$$

where $\tilde{p}_i(\boldsymbol{k},z)$ is the solution of the depth-dependent wave equation in layer i. The total contribution from an infinite plane of random monopoles located at $z = z_s$ is then determined by integration over the source plane, yielding

$$p_i(\boldsymbol{r}, z; z_s) = \frac{1}{2\pi}\iint d^2\boldsymbol{r}_s d^2\boldsymbol{k} S_\omega(\boldsymbol{r}_s, z_s)\tilde{p}_i(\boldsymbol{k}, z)e^{-i\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}_s)}. \quad \mathbf{(10)}$$

It is noted that S_ω is a random variable, so as $\tilde{p}_i(\boldsymbol{k},z)$ in the layer that is random.

The solution for the pressure field in the random medium can also be represented by a mean field and a scattered field as shown in Eq. (4), which by taking the Fourier transform, becomes

$$\tilde{p}_i = \left\langle \tilde{p}_i \right\rangle_{c_i'} + \tilde{p}_i^s, \quad \mathbf{(11)}$$

where the subscript c' stands for ensemble averaging over the random medium. Substituting Eq. (11) into Eq. (10), and then taking the ensemble average of the product of $p_i(\boldsymbol{r}_1, z_1; z_s)$ and $p_i^*(\boldsymbol{r}_2, z_2; z_s)$, with * standing for the complex conjugate, we can obtain the cross-correlation function of the noise field:

$$\begin{aligned} C(\boldsymbol{r}_1, \boldsymbol{r}_2, z_1, z_2) &= \left\langle p_i(\boldsymbol{r}_1, z_1; z_s)p_i^*(\boldsymbol{r}_2, z_2; z_s)\right\rangle \\ &= \frac{1}{(2\pi)^2}\iiiint d^2\boldsymbol{r}_s' d^2\boldsymbol{r}_s'' d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2 \\ &\quad \times e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{r}_1-\boldsymbol{r}_s')}e^{-i\boldsymbol{k}_2\cdot(\boldsymbol{r}_2-\boldsymbol{r}_s'')}\Big\langle S_\omega(\boldsymbol{r}_s')S_\omega^*(\boldsymbol{r}_s'') \\ &\quad \times \left[\left\langle \tilde{p}_i(\boldsymbol{k}_1,z_1)\right\rangle_{c_i'} + \tilde{p}_i^s(\boldsymbol{k}_1,z_1)\right] \\ &\quad \times \left[\left\langle \tilde{p}_i(\boldsymbol{k}_2,z_2)\right\rangle_{c_i'} + \tilde{p}_i^s(\boldsymbol{k}_2,z_2)\right]^*\Big\rangle . \end{aligned} \quad \mathbf{(12)}$$

To proceed, the expression inside the angle bracket $\langle \cdots \rangle$ can be expanded as

$$\begin{aligned} &\Big\langle S_\omega(\boldsymbol{r}_s')S_s^*(\boldsymbol{r}_s'')\left[\left\langle \tilde{p}_i(\boldsymbol{k}_1,z_1)\right\rangle_{c_i'} + \tilde{p}_i^s(\boldsymbol{k}_1,z_1)\right] \\ &\quad \times \left[\left\langle \tilde{p}_i(\boldsymbol{k}_2,z_2)\right\rangle_{c_i'} + \tilde{p}_i^s(\boldsymbol{k}_2,z_2)\right]^*\Big\rangle \\ &= \left\langle S_\omega(\boldsymbol{r}_s')S_\omega^*(\boldsymbol{r}_s'')\right\rangle\left\langle \tilde{p}_i(\boldsymbol{k}_1,z_1)\right\rangle_{c_i'}\left\langle \tilde{p}_i(\boldsymbol{k}_2,z_2)\right\rangle_{c_i'}^* \\ &\quad + \left\langle S_\omega(\boldsymbol{r}_s')S_\omega^*(\boldsymbol{r}_s'')\tilde{p}_i^s(\boldsymbol{k}_1,z_1)\tilde{p}_i^{s,*}(\boldsymbol{k}_2,z_2)\right\rangle \\ &\quad + \left\langle S_\omega(\boldsymbol{r}_s')S_\omega^*(\boldsymbol{r}_s'')\left\langle \tilde{p}_i(\boldsymbol{k}_1,z_1)\right\rangle_{c_i'}\tilde{p}_i^{s,*}(\boldsymbol{k}_2,z_2)\right\rangle \\ &\quad + \left\langle S_\omega(\boldsymbol{r}_s')S_\omega^*(\boldsymbol{r}_s'')\tilde{p}_i^s(\boldsymbol{k}_1,z_1)\left\langle \tilde{p}_i(\boldsymbol{k}_2,z_2)\right\rangle_{c_i'}^*\right\rangle . \end{aligned} \quad \mathbf{(13)}$$

Based on the solution of the scattered field shown in Eq. (7), and with the assumptions that the random noise sources S_ω are statistically independent of the random sound speed variations ε_i in the medium, the last two terms in Eq. (13) vanish due to the zero mean assumption (i.e., $\langle \varepsilon_i \rangle = 0$). As a result, the cross-correlation function becomes

$$\begin{aligned} &C(\boldsymbol{r}_1, \boldsymbol{r}_2, z_1, z_2) \\ &= \frac{1}{(2\pi)^2}\iiiint d^2\boldsymbol{r}_s' d^2\boldsymbol{r}_s'' d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2 e^{-i\boldsymbol{k}_i\cdot(\boldsymbol{r}_1-\boldsymbol{r}_s')}e^{i\boldsymbol{k}_2\cdot(\boldsymbol{r}_2-\boldsymbol{r}_s'')} \\ &\quad \times \Big[\left\langle S_\omega(\boldsymbol{r}_s')S_\omega^*(\boldsymbol{r}_s'')\right\rangle\left\langle \tilde{p}_i(\boldsymbol{k}_1,z_1)\right\rangle_{c_i'}\left\langle \tilde{p}_i(\boldsymbol{k}_2,z_2)\right\rangle_{c_i'}^* \\ &\quad + \left\langle S_\omega(\boldsymbol{r}_s')S_\omega^*(\boldsymbol{r}_s'')\tilde{p}_i^s(\boldsymbol{k}_1,z_1)\tilde{p}_i^{s,*}(\boldsymbol{k}_2,z_2)\right\rangle\Big] . \end{aligned} \quad \mathbf{(14)}$$

The first and second terms in the square bracket ([···]) are, respectively, the correlation functions for the mean field and scattered field.

In the present analysis, we shall focus on the effect of medium inhomogenieties on the spatial correlation of the mean noise field. In this regard, it is noted that the term representing the correlation of the scattered field is a second-order term; therefore, it is suppressed in this study. To further simplify the analysis, it is assumed that the random noise sources are wide-sense stationary, meaning that the spatial correlation depends upon the their separation, not on their absolute positions. Therefore, letting $\overline{\boldsymbol{r}} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ and $\overline{\boldsymbol{r}}_s = \boldsymbol{r}_s' - \boldsymbol{r}_s''$, denoting

$S_\omega(\boldsymbol{r}_s')S_\omega(\boldsymbol{r}_s'')$ as $\langle S_\omega^2\rangle N(\overline{\boldsymbol{r}}_s)$, substituting the above definition into Eq. (14), and then integrating over $\boldsymbol{r}''$ and $\boldsymbol{k}_2$, we can obtain

$$C(\overline{\boldsymbol{r}},z_1,z_2)=\langle S_\omega^2\rangle\int\int d^2\overline{\boldsymbol{r}}\,d^2\boldsymbol{k}N(\overline{\boldsymbol{r}}_s)e^{i\boldsymbol{k}\cdot(\overline{\boldsymbol{r}}-\overline{\boldsymbol{r}}_s)}$$

$$\times\langle\tilde{p}_i(\boldsymbol{k},z_1)\rangle_{c_i'}\langle\tilde{p}_i(\boldsymbol{k},z_2)\rangle_{c_i'}^*. \quad \textbf{(15)}$$

If we define the spatial correlation of the noise sources as the Fourier transform of the spectrum

$$N(\overline{\boldsymbol{r}}_s)=\frac{1}{2\pi}\int d^2\boldsymbol{k}P(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\overline{\boldsymbol{r}}_s}, \quad \textbf{(16)}$$

then Eq. (15) can be simplified as

$$C(\overline{\boldsymbol{r}},z_1,z_2)=\langle p_i(\boldsymbol{r}_1,z_1;z_s)p_i^*(\boldsymbol{r}_2,z_2;z_s)\rangle$$

$$=2\pi\langle S_\omega^2\rangle\int d^2\boldsymbol{k}P(\boldsymbol{k})$$

$$\times\langle\tilde{p}_i(\boldsymbol{k},z_1)\rangle_{c_i'}\langle\tilde{p}_i(\boldsymbol{k},z_2)\rangle_{c_i'}^*e^{i\boldsymbol{k}\cdot\overline{\boldsymbol{r}}}. \quad \textbf{(17)}$$

It is noted that, for the case of isotropic random noise sources, the power spectrum is only a function of $|\boldsymbol{k}|$. Furthermore, if the random sound-speed variations can be decomposed into a product of vertical and horizontal components, with the horizontal randomness being homogeneous so that it is isotropic and wide-sense stationary in the horizontal direction, then $\langle\tilde{p}_i(\boldsymbol{k},z_1)\rangle_{c_i'}$ is also angular independent. Under these assumptions, Eq. (17) can be reduced to a one-dimensional integral:

$$C(\overline{r},z_1,z_2)$$

$$=4\pi^2\langle S_\omega^2\rangle\int_0^\infty P(k_r)$$

$$\times\langle\tilde{p}_i(k_r,z_1)\rangle_{c_i'}\langle\tilde{p}_i(k_r,z_2)\rangle_{c_i'}^*J_0(k_r\overline{r})k_rdk_r. \quad \textbf{(18)}$$

By setting $\overline{r}=0$ and $z_1=z_2=z$, an integral leading to a quantity proportional to the noise intensity can be obtained as

$$I(z)=4\pi^2\langle S_\omega^2\rangle\int_0^\infty P(k_r)\left|\langle\tilde{p}_i(k_r,z)\rangle_{c_i'}\right|^2k_rdk_r. \quad \textbf{(19)}$$

III. Numerical Examples

In this section, we shall apply the above formulations to demonstrate the effect of medium inhomogenieties on the noise field. In view of Eqs. (17) – (19), we must first derive the solution of the mean field in the random medium. Despite the fact that Eq. (17) is capable of computing the correlation of the noise field generated by any random sources describable by a power spectrum, its efficiency heavily depends on the complexities of the solution in the random medium.

1. Solution of the Mean Field in the Random Medium

For initial analysis, we shall assume that the noise sources are white so that $P(k)$ = constant, and that the overall environment is a uniform water column overlying a fluid half-space with small random sound-speed perturbations, i.e., $c_1'=0$ and $c_2=\langle c_2\rangle+c_2'$. Moreover, we shall also assume that the spatial correlation of the sound-speed perturbation is separable and satisfies the following relation (Ivakin and Lysanov, 1981; Yamamoto, 1989):

$$\langle\varepsilon_2(\boldsymbol{R}_1)\varepsilon_2(\boldsymbol{R}_2)\rangle=4\sigma^2N(\overline{r})M(\overline{z}), \quad \textbf{(20)}$$

where σ represents the RMS randomness of $c_2'/\langle c_2\rangle$, and $\overline{r}=|\boldsymbol{r}_1-\boldsymbol{r}_2|$, $\overline{z}=|z_1-z_2|$; $N(\overline{r})$ and $M(\overline{z})$ are, respectively, the horizontal and vertical correlation functions of the random sound-speed variations. In view of its simplicity, the horizontal correlation is considered to be a Gaussian function, i.e., $N(\overline{r})=e^{-\overline{r}^2/L_0^2}$, with L_0 being the horizontal correlation distance; furthermore, $M(\overline{z})$ is considered to be δ-correlated, i.e., $M(\overline{z})=z_0\delta(\overline{z})$, so that it measures the degree of randomness in the vertical direction.

With the correlation function given by Eq. (20) and the Green's function for the half-space represented as

$$G_2(\boldsymbol{R};\boldsymbol{R}')=-\int_0^\infty\frac{1}{ik_{z,2}}\left[e^{-ik_{z,2}|z-z'|}+R_{21}e^{-ik_{z,2}|z'|-ik_{z,2}z}\right]$$

$$\times J_0(k_rr)k_rdk_r, \quad \textbf{(21)}$$

Tang and Frisk (1991) have shown that the solution for the coherent field can be expressed as

$$\langle\tilde{p}_2(k_r,z)\rangle_{c_i'}=A_2^+(k_r)\left[e^{-i\eta z}+\frac{e^{-i\eta z}}{2i\eta}\left(\int_0^z f(\xi)d\xi\right)\right.$$

$$\left.+\frac{e^{i\eta z}}{2i\eta}\left(\int_z^\infty f(\xi)e^{-i2\eta\xi}d\xi\right)\right], \quad \textbf{(22)}$$

where

$$f(\xi)=2\langle k_2\rangle^4\sigma^2z_0\int_0^\infty\frac{iR_{21}}{k_{z,2}}e^{-i2k_{z,2}\xi}H(k,k_r)kdk, \quad \textbf{(23)}$$

$$H(k,k_r)=\frac{1}{2}L_0^2e^{-(k_r^2+k^3)L_0^2/4}I_0(k_rkL_0^2/2), \quad \textbf{(24)}$$

$$R_{21} = \frac{\rho_1 k_{z,2} - \rho_2 k_{z,1}}{\rho_1 k_{z,2} + \rho_2 k_{z,1}}, \tag{25}$$

$$\eta^2 = \langle k_2 \rangle^2 \left[1 - \frac{k_r^2}{\langle k_2 \rangle^2} + c(k_r) \right], \tag{26}$$

$$c(k_r) = 2\langle k_2 \rangle^2 \sigma^2 z_0 \int_0^\infty \frac{i}{k_{z,2}} H(k, k_r) k dk, \tag{27}$$

$$k_{z,2} = \sqrt{\langle k_2 \rangle^2 - k_r^2}. \tag{28}$$

The parameter $A_2^+(k_r)$ is a constant, representing the amplitude of the down-going wave. The solution is valid for the case in which $z_0 << \lambda$, where λ is the acoustic wavelength.

With the solution of the coherent field in the lower half-space given by Eq. (22), and with the solution in the water column readily derived as

$$\tilde{p}_1(k_r, z) = A_1^+(k_r) e^{-ik_{z,1}z} + A_1^-(k_r) e^{ik_{z,1}z} + \frac{e^{-ik_{z,1}|z-z_s|}}{4\pi i k_{z,1}}, \tag{29}$$

where $k_{z,1} = \sqrt{k_1^2 - k_r^2}$ and $A_1^\pm(k_r)$ are unknown constants, a linear system can be established by invoking the boundary conditions of the continuities of pressure and vertical displacement:

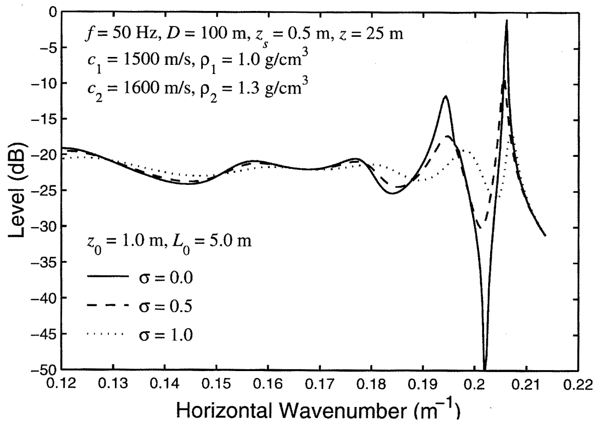

Fig. 2. Wavenumber spectrum.

values presented in the figures, such as the mean-noise levels, are not compared with the experimental data, a procedure requiring much more extensive analysis, which is beyond the scope of this study (Schmidt and Kuperman, 1988).

The wavenumber spectrum of the noise field reveals the spectral contents in the noise field and can be presented by plotting $\log\left|\langle \tilde{p}_i(k_r, z) \rangle\right|$ versus k_r. Figure 2 shows the wavenumber spectrum for a frequency of 50 Hz in a 100 m water column; all the other parameters are explained in the legend. The curves in the figure demonstrate the variations of the

$$\begin{bmatrix} 1 & 1 & 0 \\ e^{-ik_{z,1}D} & e^{ik_{z,1}D} & -1 - \frac{1}{2i\eta}\int_0^\infty f(\xi) e^{-2i\eta\xi} d\xi \\ -\frac{ik_{z,1}}{\rho_1\omega^2} e^{-ik_{z,1}D} & \frac{ik_{z,1}}{\rho_1\omega^2} e^{ik_{z,1}D} & \frac{1}{\rho_2\omega^2}\left(i\eta - \frac{1}{2}\int_0^\infty f(\xi) e^{-2i\eta\xi} d\xi \right) \end{bmatrix} \times \begin{bmatrix} A_1^-(k_r) \\ A_1^+(k_r) \\ A_2^+(k_r) \end{bmatrix} = -\frac{1}{4\pi i} \begin{bmatrix} \frac{1}{k_{z,1}} e^{-ik_{z,1}|z_s|} \\ \frac{1}{k_{z,1}} e^{-ik_{z,1}|D-z_s|} \\ -\frac{i}{\rho_1\omega^2} e^{-ik_{z,1}|D-z_s|} \end{bmatrix}. \tag{30}$$

The above linear system can be solved for each value of k_r, leading to solutions for $\tilde{p}_1$ and $\langle \tilde{p}_2 \rangle$ and other derived quantities of interest.

IV. Results and Discussion

To reveal the properties of the noise field under the influence of sound-speed perturbations, we shall consider the wavenumber spectrum, noise intensity, and spatial correlation. It is stressed here that the numerical results generated in this study are meant to demonstrate the *qualitative* effects of sound-speed perturbation on the noise field; therefore, the absolute

spectral levels in the water column for various degrees of randomness measured based on the parameter σ. It is seen that between $\langle k_2 \rangle = \omega/\langle c_2 \rangle$ = 0.196 m^{-1} and $k_1 = \omega/c_1$ = 0.209 m^{-1}, there exist two normal modes, with the second mode (right most peak) showing greater strength (a higher level and sharper peak) in this case. For k_r less than $\langle k_2 \rangle$, it is the continuous regime, representing wave components interacting with the interface with a gazing angle higher than the critical angle, so that the wave continues to lose energy into the lower medium when it propagates through the waveguide. Figure 2 shows that in the continuous spectral regime, the curves are roughly at the same level, indicating that the randomness in

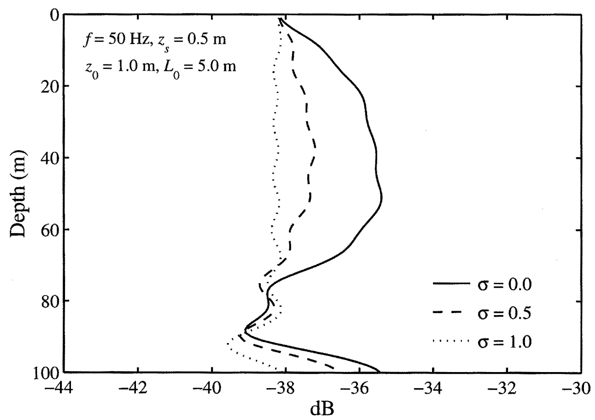

Fig. 3. Intensity distribution.

the lower medium has little effect on these wave components. However, in the normal mode regime, these curves show a discernible discrepancy; in particular, both modes becomes less prominent as they become lower and wider. It is also noted that the modal wavenumbers are slightly displaced due to the change of the medium property. The decrease in modal strength occurs due to the fact that scattering in the lower medium has an effect resembling that of medium absorption so that, despite the impingment of the wave on the interface with an incident angle shallower than the critical angle, energy penetrates into the lower medium through volume scattering, making the characteristics of the normal modes less prominent.

Next, we will consider the noise intensity. Equation (19) presents the formulas for noise intensity at depth z. It indicates that the noise intensity is only a function of the depth coordinate and is independent of the range. Furthermore, the integral shows that the total intensity is a direct integration of the wavenumber spectrum weighted over the random noise spectrum, in contrast to the computation of the transmission loss as a function of the range for a discrete source, where the integration is heavily influenced by phase interference. Therefore, in the present case, all the wave components, including the continuous spectrum and normal modes, are important, which is unlike the case with discrete sources, in which the normal modes are the mechanisms dominating waveguide propagation. Figure 3 shows the intensity distribution inside the water column for various values of σ. The results clearly indicate that the intensity is affected by the medium inhomogenieties, with the amount of reduction in intensity increasing with the degree of randomness measured based on the parameter σ. Again, this is due to leakage of energy into the lower medium through volume scattering.

Finally, the spatial correlation of the noise field will be considered. The correlation function, Eq. (17), characterizes the spatial statistics of the noise field through its magnitude and decay rate. Figure 4 shows the magnitude of the horizontal correlation of the noise field for the lower medium with and without sound-speed perturbations, represented, respectively, by the solid and dashed curves. The results demonstrate that the magnitude of the correlation in the water column decreases if the medium is subject to random sound-speed perturbations, indicating that the energy in the coherent field is extracted and dispersed into the scattered field.

To observe the characteristics of the noise field in terms of the relative relationship, a normalized correlation function with respect to zero-separation can be employed. Figure 5 shows the normalized correlation of the noise field for three values of the correlation length of the random medium L_0. The results show that the larger the value of L_0, the more slowly the corresponding curve decays, demonstrating that the degree of incoherence of the noise field increases with that of the medium inhomogenieties. This is consistent with our general perception that the randomness of the noise field is partly attributable to that of the random medium.

V. Conclusions and Remarks

In this analysis, we have studied the effects of medium inhomogenieties due to sound-speed perturbations on surface-

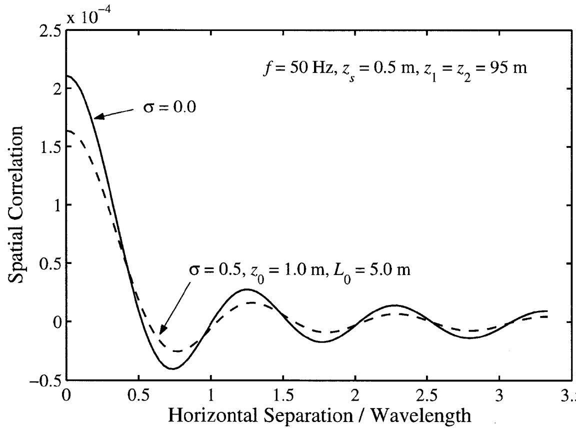

Fig. 4. Horizontal spatial correlation of the noise field.

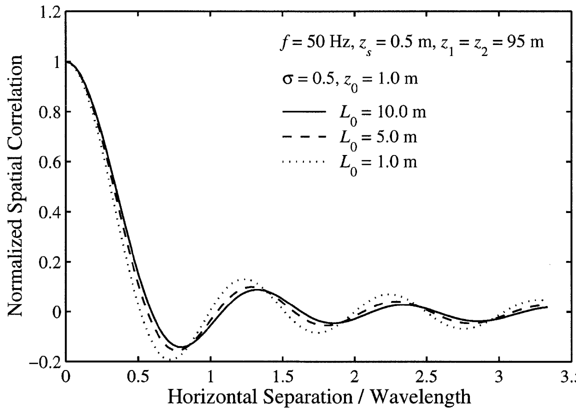

Fig. 5. Normalized horizontal spatial correlation for various correlation length of random medium.

generated ambient noise. By applying a formulation similar to a previously developed noise-generation model with consideration of medium randomness, we have been able to analyze the characteristics of the noise field, including its intensity and spatial correlation.

The results have demonstrated that the medium inhomogenieties reduce the efficiency of waveguide propagation because volume scattering serves as a mechanism for transmitting energy out of waveguide, which in turn reduces the strength of the normal modes. As a result, the coherent energy inside the waveguide decreases according to the degree of randomness in the medium, which is equivalent to the effect of medium absorption. Moreover, the spatial coherence of the noise field has been studied, and it has been found that the decay rate of the spatial correlation function increases with a decrease of the medium correlation length, indicating that the coherence of the noise field decreases as the medium becomes more random.

It is noted that, since we have only conducted leading-order analysis in this study, some features embedded in the scattered field are not found in the results, in particular the spatial structure of the noise field. However, the noise intensity, which in this case is not subject to any phase-interference effect [see Eq. (19)], should be less susceptible to the higher-order effect. In view of the fact that the spatial correlation of the noise field depends upon the source spectrum, medium randomness, and waveguide regularity, complete analysis of the noise field remains for future work.

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隨機不均勻介質對海面所產生之環境噪聲的影響

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摘　要

本文在於探討由於隨機聲速分佈所造成介質之不均勻性，對海面所產生之環境噪聲的影響。藉由噪聲產生模式與隨機不均勻介質中聲波傳播原理，本文推導出平均噪聲模式，並應用於典型的海洋環境以便探討噪聲場性質，包括波數譜、噪聲聲強分佈、空間關連性等。本文結果顯示，隨機不均勻聲速分佈對聲場所造成的影響，相當於具有吸收性介質的效應，因此，使得波導中之簡正模態變得較不顯著，也隨即降低了噪聲聲強的大小。另一方面，噪聲場的空間均致性也會隨著隨機聲速的亂度增加而減小，顯示環境噪聲場之空間關連性亦會受到不均勻聲速分佈的影響。