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Analysis of Structures with Spatial Variation of Material Property

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ABSTRACT

The reliability of structures in the presence of uncertainty has been a crucial factor in their analysis and design. A typical example of system stochasticity problems is the spatial variation of the material property. Indeed, the primary focus of this study was to derive the stochastic stiffness matrices for a truss and a beam element, so that the conventional finite element method could be utilized to evaluate the response variability of a structure whose material property exhibited spatial random variation.

A structural element is divided into several sub-elements whose number depends on the scale of fluctuation in a random field. In each sub-element, the elastic characteristics are represented by the local "spatial average" of the field. Upon application of static condensation, we eliminate the additional degrees-of-freedom stemming from the division of sub-elements to produce a new element stiffness matrix whose size is equal to the conventional one. The global stiffness matrix and system equation are then established. With the aid of the first-order perturbation method, we obtain the nodal displacements, which consist not only of the deterministic component, but also of the random component. Numerical examples show that this study provides a more tractable procedure for determining the response variability of a complex structure, with good accuracy and computational efficiency.

Key Words: material variation, finite element method, stochastic process

I. Introduction

Modern structural systems require stringent reliability standards. The reliability of structures in the presence of uncertainty has been a crucial factor in their analysis and design. A typical example of system stochasticity problems is the spatial variation of the mechanical properties of materials, such as the Young's modulus and failure strength. In recent years, this has been an issue of great interest to many researchers. A small number of analytic solutions to such problems are available, mainly for simple structures (Shinozuka, 1987). Although the same concept can be extended even further to deal with more complicated structures, the associated analytical formulas become rather cumbersome (Kardara *et al.*, 1989). Therefore, the majority of research work in this area has focused on developing various stochastic finite element methods (SFEM) to obtain solutions numerically.

In an earlier work on SFEM (Handa and Anderson, 1981), material properties were deemed as random variables, and structures could be dissected in such a manner that the essential variation of material parameters could be considered. In a more practical representation, the elastic modulus was regarded as a stochastic process (Hisada and Nakagiri, 1980, 1985; Deodatis, 1990a, 1990b). The stochastic stiffness matrix was obtained by (1) using the form of the conventional stiffness matrix with the elastic modulus being replaced by a sum of a deterministic term and a random one, or (2) adding the random parts of the stiffness matrix to the conventional stiffness matrix. These measures may not be exact since, due to the material variation within an element, the conventional shape function cannot be employed to derive the stochastic stiffness matrix (Dasgupta and Yip, 1989).

Other works on the same subject are also found in the literature. One study found that the continuous random field could be taken care of by means of local or weighted integrations to construct the element stiffness matrix (Takada, 1990). In consequence, the random field is transformed into a problem involving only a few random variables. This leads to an improvement in computational efficiency. An alternate method for computing the component of the stochastic element stiffness was proposed by Spanos and Ghanem (1989). Their work was based on the orthogonal expansion of a stochastic process. The expansion consists of the projection of the process onto a space of orthogonal random variables. To apply the method, however, it is necessary to determine, analytically or even numerically, the eigenvalues and eigenvectors of the associated convariance function.

In view of the lastest developments in SFEM, this study intends to provide a more systematic method for evaluating the response variability of a structure whose Young's modulus is assumed to constitute a homogeneous one-dimensional and univariate stochastic process. In this connection, the stochastic element stiffness matrices of a truss and a beam are derived. An element is divided into several sub-elements whose number depends on the scale of fluctuation of the process. In any sub-element, the elastic characteristics are represented by the local "spatial average" of the process (Vanmarcke and Grigoriu, 1983; Zhu *et al.*, 1992). Consequently, the stiffness matrix of the sub-element is obtained. Using the technique of static condensation, we eliminate the additional degrees-of-freedom stemming from the division of sub-elements to produce a new element stiffness matrix whose size is equal to the conventional one. The global stiffness matrix is then established by the well-known direct stiffness method. The stiffness and the displacement matrices in the system equation are expanded with respect to the probabilistic variables which reflect the material spatial variations, i.e., local averages of all sub-elements. Upon application of the first-order perturbation method (Baecher and Ingra, 1981; Hisada and Nakagiri, 1981), the nodal displacements are determined. The covariance of responses is a function of the covariance of the local spatial averages. When an appropriate variation function, which depicts the dependence of covariance of the spatial averages on the average interval size, is selected, all the random terms involved in the response variability are obtained.

II. Truss Element

Consider the truss element in Fig. 1 with a deterministic axial load and having a modulus of elasticity varying randomly along its length, which is given by

$$E(x) = \overline{E}\,(1 + g(x)), \tag{1}$$

where $E(x)$ and $g(x)$ are one-dimensional homogeneous stochastic processes, and $\overline{E}$ is $\varepsilon[E(x)]$. Note that $\varepsilon[\bullet]$ is the expectation. $\overline{E}\,g(x)$ represents the deviation of the modulus of elasticity around its mean value. The function $g(x)$ is a zero-mean stochastic field with autocorrelation function $R_{gg}(\xi)$.

The conventional derivation of the stiffness matrix

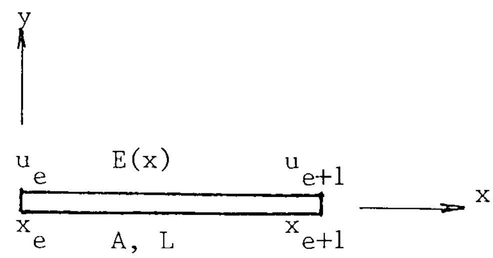

Fig. 1. A truss element.

starts with the assumption of a displacement function, i.e.,

$$u(x)=N_e u_e+N_{e+1}u_{e+1}, \tag{2}$$

in which the shape functions N_e and N_{e+1} are given by

$$N_e=(x_{e+1}-x)/L \quad N_{e+1}=(x-x_e)/L. \tag{3}$$

However, in the present case, Eqs. (2) and (3) are not appropriate because of the variation of the elasticity modulus.

To tackle this problem, we introduce the concept of a local average. The i-th local average of $g(x)$ is defined as

$$\overline{g}_i(x_i) = \frac{1}{l}\int_{x_i-\frac{l}{2}}^{x_i+\frac{l}{2}} g(x)\,dx, \tag{4}$$

where l=length of the local average, and x_i=centroid of the length. It is shown that

$$\varepsilon[\,\overline{g}_i(x_i)\,]=0 \quad Var[\,\overline{g}_i(x_i)\,]=\sigma_g^2\bullet\gamma(l), \tag{5}$$

where σ_g^2 is the variance of $g(x)$, and $\gamma(l)$ is the variance function of $g(x)$, which measures the reduction of the point variance, σ_g^2, under local averaging. The variance function is related to the autocorrelation function as follows:

$$\gamma(l)=\frac{2}{l}\int_0^l \left(1-\frac{\xi}{l}\right) R_{gg}(\xi)\,/\,\sigma_g^2\,d\xi. \tag{6}$$

Depending upon the scale of fluctuation of the process, this element is further divided into N sub-elements. The elasticity modulus in each sub-element may be represented by a local average of $E(x)$, $\overline{E}_i$, which is a specific value of a random variable. By this measure, the shape functions similar to Eq. (3) can be used. Note that L in Eq. (3) is replaced by l ($l=L/N$) under this condition.

The stiffness matrix of the i-th sub-element in the

e-th element is

$$[\bar{K}_{ei}]=\int_0^l \begin{bmatrix}-1/l\\1/l\end{bmatrix} \bar{E}_i \begin{bmatrix}-1/l\\1/l\end{bmatrix}^t A\,dx$$

$$=\frac{\bar{E}A}{l}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}+\frac{\bar{E}A}{l}\bar{g}_i\begin{bmatrix}1&-1\\-1&1\end{bmatrix}. \qquad (7)$$

The first and the second terms in Eq. (7) are the deterministic (conventional) and random components, respectively.

When N=2, the assembled element stiffness matrix is

$$\frac{\bar{E}A}{l}\begin{bmatrix}C_1 & -C_1 & 0\\ -C_1 & C_1+(1+\bar{g}_2) & -(1+\bar{g}_2)\\ 0 & -(1+\bar{g}_2) & (1+\bar{g}_2)\end{bmatrix},$$

in which $C_1=1+\bar{g}_1$. Using the technique of static condensation, we eliminate the internal degrees-of-freedom to yield a new element stiffness matrix, which has the same size as does the conventional one. The outcome is

$$[\bar{K}_e]=\frac{\bar{E}A}{l}C_2\begin{bmatrix}1&-1\\-1&1\end{bmatrix} \quad C_2=\frac{C_1(1+\bar{g}_2)}{C_1+(1+\bar{g}_2)}. \qquad (8)$$

It is not difficult to show that the general form of the above equation can be written as

$$[\bar{K}_e]=\frac{\bar{E}A}{l}C_N\begin{bmatrix}1&-1\\-1&1\end{bmatrix} \quad C_N=\frac{C_{N-1}(1+\bar{g}_N)}{C_{N-1}+(1+\bar{g}_N)}. \qquad (9)$$

III. Perturbation Method

When all the element stiffness matrices are determined, we combine them to find the global stiffness matrix. After the boundary condition is imposed, the equilibrium equation is

$$[K][U]=[P]_0, \qquad (10)$$

in which $[K]$ is the stiffness matrix, $[U]$ is the free or unknown displacement vector, and $[P]_0$ is the external force vector. It should be noted that $[K]$ involves a set of random components, i.e., all the local averages in each element.

Equation (10) can be solved by using the first-order perturbation method. The stiffness matrix is expanded in a Taylor series about the mean value of local average $\bar{g}_{ei}$. The symbol $\bar{g}_{ei}$ is used in place of $\bar{g}_i$ in order to indicate that this local average is associated with the e-th element. When the first-order approximation is adopted, we have

$$[K]=[K]|_E+\sum_e^{Ne}\sum_i^N \frac{\partial[K]}{\partial \bar{g}_{ei}}\Big|_E \bullet \bar{g}_{ei}. \qquad (11)$$

The displacement vector is expanded in the same way Substituting two expansions into Eq. (10) and simplifying them, we have (Lu, 1991)

$$[U]=[U]_0-\sum_e^{Ne}\sum_i^N [K]_0^{-1}[K]_0^{(e)}[U]_0 \bullet \bar{g}_{ei}/N, \qquad (12)$$

in which $[U]_0$=mean displacement vector, i.e., without consideration of the random property; $[K]_0$= conventional stiffness matrix; and $[K]_0^{(e)}$ is a sub-matrix of $[K]_0$ and consists of the components contributed by the e-th element.

The covariance of Eq. (12) is obviously a function of the covariance of the local averages, which is expressed as (Vanmarcke and Grigoriu, 1983)

$$Cov(\bar{g}_{ei},\bar{g}_{fj})$$

$$=\frac{1}{2}\frac{\sigma_g^2}{l_e l_f}[B_0^2\gamma(B_0)-B_1^2\gamma(B_1)+B_2^2\gamma(B_2)-B_3^2\gamma(B_3)], \qquad (13)$$

where B_i are the local distances defined in Fig. 2. Therefore, with a specific autocorrelation function, $R_{gg}(\xi)$, of the stochastic field, we can calculate the covariance of displacements upon application of Eqs. (6) and (13).

Similarly, on the basis of first-order approximation, the stress vector of the f-th element is

$$[\bar{S}_f]=[\bar{S}_f]_0+\sum_i^N([\bar{K}_f]_0\,\bar{g}_{fi}/N)[T_f][U]_0$$

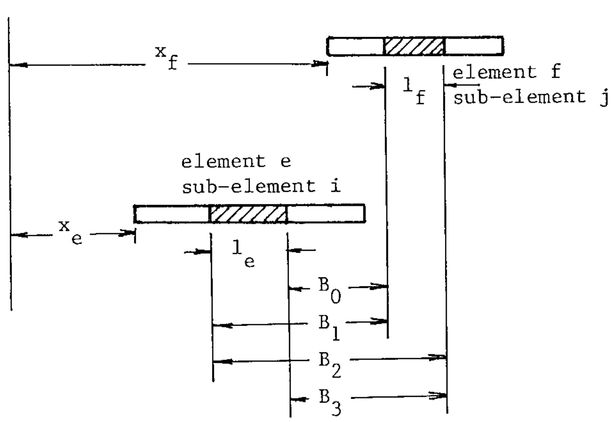

Fig. 2. Definition of local distance.

$$-\sum_{e}^{Ne}\sum_{i}^{N}[\overline{K}_f]_0\,[T_f]\,(\,[K]_0^{-1}\,[K]_0^{(e)}\,[U]_0\bullet\overline{g}_{ei}/N\,)\,, \tag{14}$$

where $[\overline{S}_f]_0$ =mean (conventional) stress vector, $[\overline{K}_f]_0$ =deterministic component of $[\overline{K}_f]$, and $[T_f]$= transformation matrix of displacement. It is not difficult to calculate the statistical properties of the stress vector in Eq. (14).

IV. Beam Element

A beam element is divided into N sub-elements. The stiffness matrix of the i-th sub-element in the e-th element is

$$[\overline{K}_{ei}]=\frac{\overline{EI}}{l^3}(1+\overline{g}_i)\begin{bmatrix}12 & 6l & -12 & 6l\\ 6l & 4l^2 & -6l & 2l^2\\ -12 & -6l & 12 & -6l\\ 6l & 2l^2 & -6l & 4l^2\end{bmatrix}, \tag{15}$$

in which I is the moment of inertia. On the basis of the condensation technique, the stochastic stiffness matrix for a beam element can be derived.

Because of its algebraic complexity, the derivation is accomplished by applying MACSYMA TM(1988). The element stiffness matrix turns out to have the following form:

$$[\overline{K}_e]=\sum_{P=1}^{N}(R_1^2\dots R_{p-1}^2R_pR_{p+1}^2\dots R_N^2)[\overline{K}_e(p)]/(D_1+D_2)\,, \tag{16}$$

in which

$$[\,\overline{K}_e(p)\,]=\frac{N^3\overline{EI}}{L^3}\begin{bmatrix}12 & & & sym\\ 6(2p-1)l & [4+12p(p-1)]l^2 & & \\ -12 & -6(2p-1)l & 12 & \\ \begin{bmatrix}12N-\\6(2p-1)\end{bmatrix}l & \begin{bmatrix}(6N-4)+\\12(N-p)(p-1)\end{bmatrix}l^2 & -\begin{bmatrix}12N-\\6(2p-1)\end{bmatrix}l & \begin{bmatrix}4+12(N-p)\\(N-p+1)\end{bmatrix}l^2\end{bmatrix} \tag{17}$$

$$R_p=1+\overline{g}_p \tag{18}$$

$$D_1=\sum_{i=1}^{N}\prod_{j=1\,j\neq i}^{N}R_j^2 \qquad N=2,\ 3,\ \dots \tag{19}$$

$$D_2=\sum_{k=1}^{N-1}\sum_{q=1}^{N-k}(12k^2+2)(R_1^2\dots R_{q-1}^2R_qR_{q+1}^2\dots R_{q+k-1}^2R_{q+k}R_{q+k+1}^2\dots R_N^2)\,. \tag{20}$$

Equations (16)-(20) can be further illustrated. For instance, if N=3, we have

$$D_1=R_2^2R_3^2+R_1^2R_3^2+R_1^2R_2^2\,. \tag{21}$$

In Eq. (20), k is an integer which indicates the seperation distance between two linear terms, i.e., R_q and R_{q+k}. When k=1, we have

$$\sum_{q=1}^{N-k}(12k^2+2)(R_1^2\dots R_N^2)=14R_1R_2R_3^2+14R_1^2R_2R_3\,.$$

When k=2, we have

$$\sum_{q=1}^{N-k}(12k^2+2)(R_1^2\dots R_N^2)=50R_1R_2^2R_3\,.$$

As a result,

$$D_2=14R_1R_2R_3^2+50R_1R_1^2R_3+14R_1^2R_2R_3\,. \tag{22}$$

If the element in Eq. (16) is denoted as $s(i,\,j)$, then

$$s(1,1)=\frac{N^3\overline{EI}}{L^3}\bullet\frac{12R_1R_2^2R_3^2+12R_1^2R_2R_3^2+12R_1^2R_2^2R_3}{D_1+D_2}\,. \tag{23}$$

Similarly,

$$s(2,1)=\frac{N^2\overline{EI}}{L^2}\bullet\frac{6R_1R_2^2R_3^2+18R_1^2R_2R_3^2+30R_1^2R_2^2R_3}{D_1+D_2}\,. \tag{24}$$

When all R_p are equal to 1, i.e., the deterministic case, Eq. (16) is nothing but the conventional stiffness of a beam element.

V. Nodal Displacement

The system stiffness matrix is associated with all the pertinent local averages in the elements. Expansion of this matrix leads to

$$[K] = [K]|_E + \sum_{e}^{Ne}\sum_{i}^{N} \frac{\partial [K]}{\partial \bar{g}_{ei}} |_E \bullet \bar{g}_{ei}$$

$$= [K]_0 + \sum_{e}^{Ne}\sum_{i}^{N} [T_e]^t \bullet \frac{\partial [\bar{K}_e]}{\partial \bar{g}_{ei}} |_E \bullet [T_e] \bullet \bar{g}_{ei} \qquad \textbf{(25)}$$

$$\frac{\partial [\bar{K}_e]}{\partial \bar{g}_{ei}} |_E = \frac{2N^4 - 2N + 2 - F(i)}{N^4} \bullet [\bar{K}_e]_0 - \frac{1}{N^4} [\, \bar{K}_e(i)\,], \qquad \textbf{(26)}$$

where $[\, \bar{K}_e(i)\,]$ is from Eq. (17), and $F(i)$ is a function of N (Lu, 1991).

The perturbation technique also produces the first-order approximations of nodal displacements and stresses:

$$[U] = [U]_0 - \sum_{e}^{Ne}\sum_{i}^{N} [K_0]^{-1} \frac{\partial [\bar{K}_e]}{\partial \bar{g}_{ei}} |_E [U]_0 \bullet \bar{g}_{ei} \qquad \textbf{(27)}$$

$$[\bar{S}_f] = [\bar{S}_f]_0 + \sum_{i}^{N} \frac{\partial [\bar{K}_f]}{\partial \bar{g}_{fi}} |_E \bullet \bar{g}_{fi} \bullet [T_f][U]_0$$

$$- \sum_{e}^{Ne}\sum_{i}^{N} [\bar{K}_f]_0 [T_f] ([K_0]^{-1} \frac{\partial [K]}{\partial \bar{g}_{ei}} |_E [U]_0 \bullet \bar{g}_{ei}). \qquad \textbf{(28)}$$

VI. Numerical Examples

Two structures, a frame and a truss, shown in Fig. 3, are analyzed. Table 1 lists five types of autocorrelation functions (AFs) and the corres ponding variance functions. The degree of fluctuation of a process is usually measured by the correlation distance, or the scale of fluctuation, which is defined as

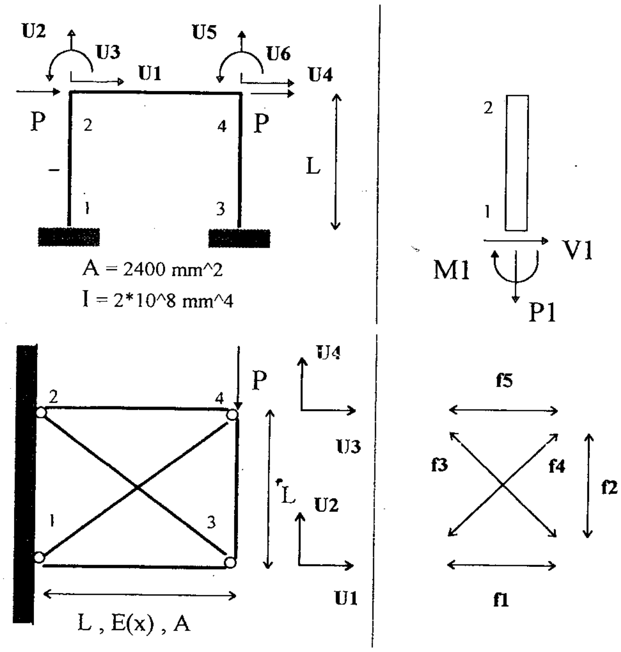

Fig. 3. Frame and truss structures.

$$\theta = \lim_{l \to \infty} l\gamma(l). \qquad \textbf{(29)}$$

It can also be obtained from

$$\theta = \int_{-\infty}^{\infty} R_{gg}(\tau)/\sigma_g^2\, d\tau. \qquad \textbf{(30)}$$

The respective correlation distance is also shown in Table 1. Under a specific correlation distance, e.g. θ=5, various AFs can be delineated as shown

Table 1. Types of Autocorrelation Functions

TYPE	$R_{gg}(\tau)$	Variance Function $\gamma(L)$	θ
A	$\sigma_g^2 \quad \lvert\tau\rvert \le b/2$ $0 \quad \lvert\tau\rvert \ge b/2$	$1 \quad \lvert L\rvert \le b/2$ $\frac{b}{\lvert L\rvert}(1 - \frac{b}{4\lvert L\rvert}) \quad \lvert L\rvert \ge b/2$	b
B	$\sigma_g^2(1 - \frac{\lvert\tau\rvert}{b})$ $\lvert\tau\rvert \le b$ 0 $\lvert\tau\rvert \ge b$	$(1 - \frac{\lvert L\rvert}{3b}) \quad \lvert L\rvert \le b$ $\frac{b}{\lvert L\rvert}(1 - \frac{b}{\lvert L\rvert}) \quad \lvert L\rvert \ge b$	b
C	$\sigma_g^2 \frac{(1 - 3(\frac{\tau}{b})^2)}{(1 + (\frac{\tau}{b})^2)^3}$	$\frac{b^2}{L^2} - \frac{3b^4}{L^2(b^2 + L^2)} + \frac{2(b^6 + b^4L^2)}{L^2(L^2 + b^2)^2}$	$\frac{b}{\sqrt{6}}$
D	$\sigma_g^2 \frac{1}{1 + (\frac{\tau}{b})^2}$	$\frac{2b}{\lvert L\rvert}\tan^{-1}(\frac{\lvert L\rvert}{b}) + \frac{b^2 \ell n(b^2)}{L^2} - \frac{b^2 \ell n(b^2 + L^2)}{L^2}$	$b\pi$
E	$\sigma_g^2 \exp(-(\frac{\tau}{b})^2)$	$\frac{b^2}{L^2}(-1 + \mathrm{Exp}(-(\frac{L}{b})^2) + \frac{L}{b}\sqrt{\pi}\,\mathrm{Erf}(\frac{\lvert L\rvert}{b}))$	$b\sqrt{\pi}$

$$\mathrm{Erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{Exp}(-t^2)\, dt$$

in Fig. 4. In the following calculation, the coefficient of variation (c.o.v) of $E(x)$ is equal to 0.1.

1. Number of Sub-Elements

The frame is analyzed first. Figure 5 shows, with different numbers of sub-elements, N, the c.o.v. of nodal displacement U_1 under autocorrelation function C. Rapid convergence is observed. To examine the effect of different random fields on the selection of N, Fig. 6 presents the N required to reach a fairly convergent solution under different types of AF. With an increase of θ, the system becomes less random. Therefore, N is smaller. In addition, it is seen that the type of function has some influence, but that this influence is not as great as that due to the non-dimensional correlation distance, θ/L.

The number of sub-elements needed to calculate the variability of the internal force, M_1, is illustrated in Fig. 7. It shows a similar tendency to that in Fig. 6 although the number required is generally larger. Both figures reveal that the number of sub-elements used in the analysis is acceptable.

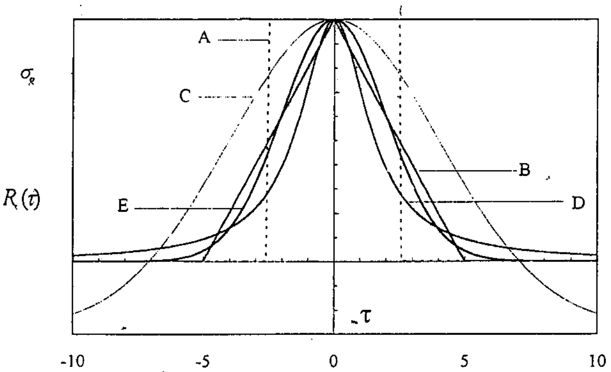

Fig. 4. Autocorrelation functions with θ=5.

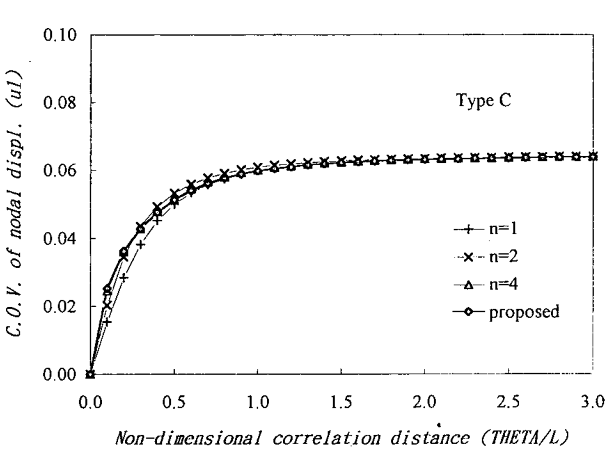

Fig. 5. c.o.v. of nodal displacement (frame).

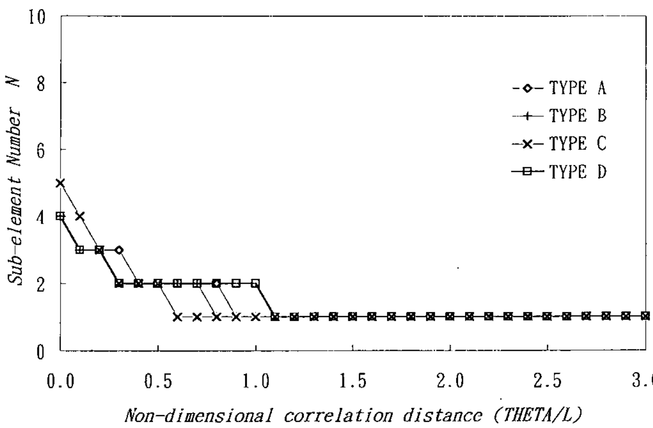

Fig. 6. Number of sub-elements required in calculating displacement variability.

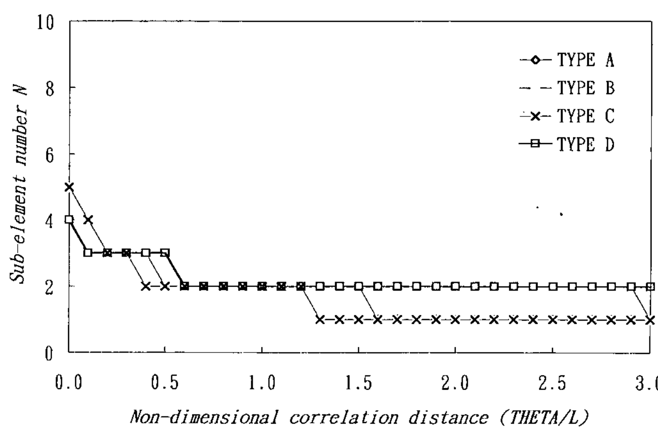

Fig. 7. Number of sub-elements required in calculating moment variability.

It is noted herein that N is always equal to one in analyzing a truss.

2. Response Variability

Figure 8 presents the c.o.v. of nodal displacement U_1 of the frame under a different AF. The displacement variability increases with θ. For a particular value of θ/L, the variability resulting from function C is slightly larger than that resulting from function D. This may be explained by Fig. 4, in which the absolute value of function C is greater than that of function D. This implies that the material variation represented by function C is more significant. As a result, the higher the correlation of material variation is, the greater the response variability is. However, it should be pointed out that the discrepancy of the various response variabilities is not more than 0.01. Thus, the effect of θ, compared to that of the type of AF, is certainly more

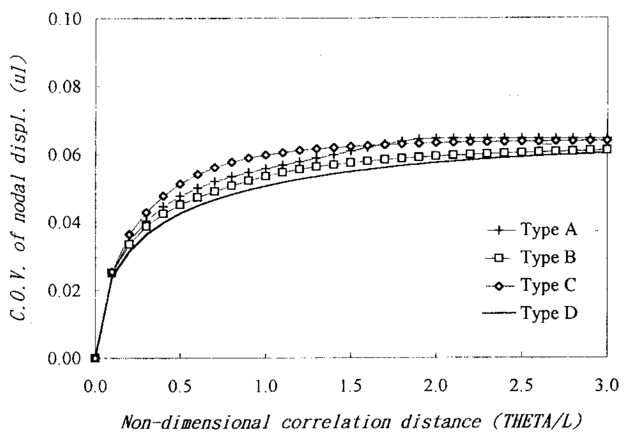

Fig. 8. Displacement variability under a different AF (frame).

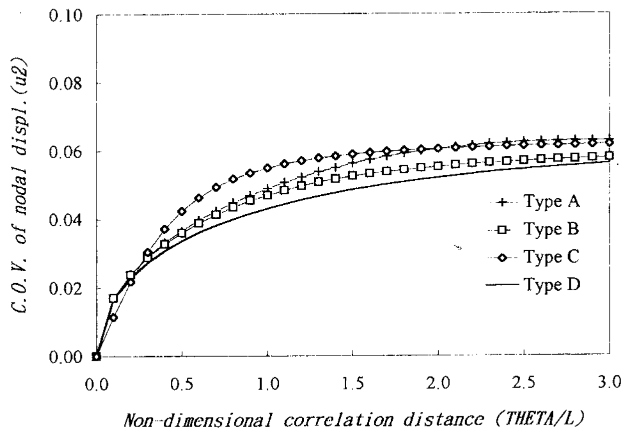

Fig. 10. Displacement variability under a different AF (truss).

decisive.

The c.o.v. of member force M_1 of the frame under a different AF is shown in Fig. 9. The similarity between this figure and Fig. 8 is obvious.

When it comes to analysis of the truss, Fig. 10 depicts the c.o.v. of nodal displacement U_2 under various AFs. The outcome is consistent with the previous discussion for the frame.

3. Accuracy

The results on the basis of the proposed stochastic stiffness matrices have been compared with those from a Monte Carlo simulation. Figures 11 and 12 show the variabilities of the nodal displacements (U_1, U_2) and the member forces (M_1, V_1, P_1) of the frame, respectively. In each figure, two methods, namely the present SFEM and the simulation method are employed. The comparison indeed shows the accuracy of the present method. It should be noted that 1000 samples were used in the simulation an-

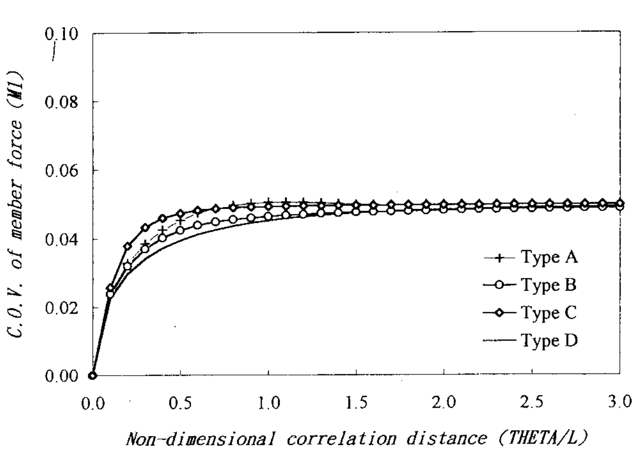

Fig. 9. Moment variability under a different AF (frame).

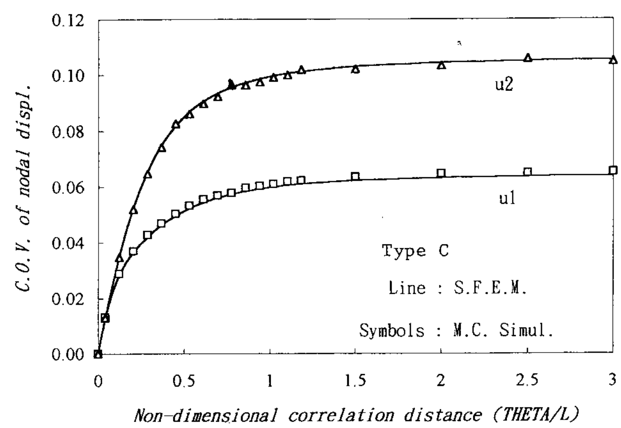

Fig. 11. Comparison of the SFEM and the simulation method (nodal displacements of the frame).

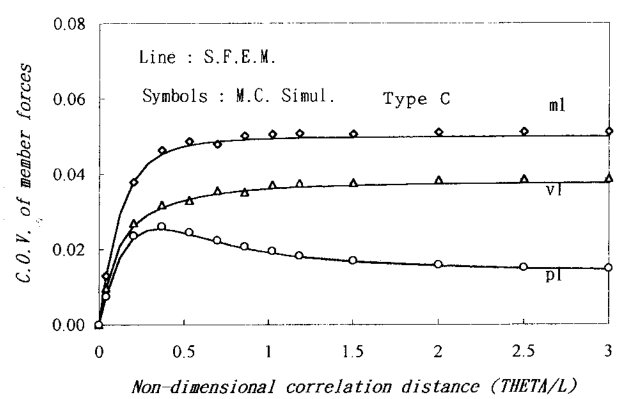

Fig. 12. Comparison of the SFEM and the simulation method (member forces of the frame).

alysis.

Figures 13 and 14 shows the c.o.v. of the nodal displacements (U_1, U_2, U_3, U_4) and the member forces (f_1, f_3) of the truss, respectively. Without the need of

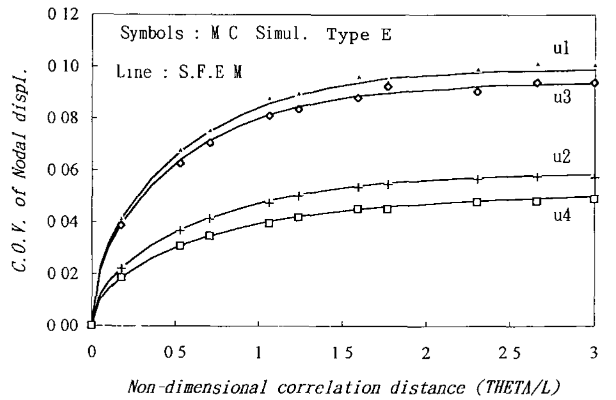

Fig. 13. Comparison of the SFEM and the simulation method (nodal displacements of the truss).

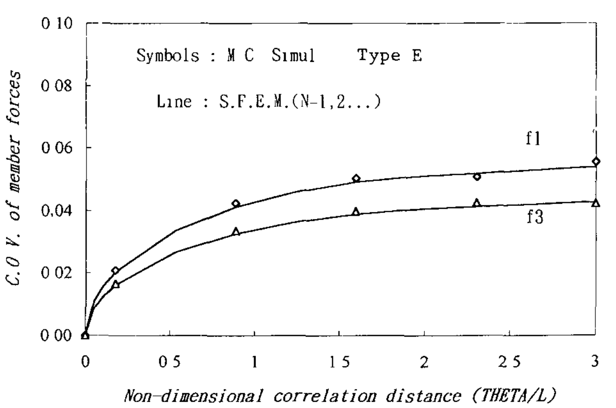

Fig. 14. Comparison of the SFEM and the simulation method (member forces of the truss).

any sub-element, i.e. N=1, the accuracy of the SFEM is revealed. In fact, the proposed SFEM is even more promising in analyzing a truss.

4. Computational Cost

The calculation was carried out on a DX2-66 PC. The computing time using different methods is summarized in Table 2. The time under column FEM refers to that required in analyzing the structures without consideration of material variation. The comparison strongly demonstrates the efficiency of the proposed SFEM in evaluating response variability.

Table 2. Computational Time of Various Methods

	FEM (sec)	SFEM (N) (sec)						Simulation Hr:Min:Sec
		1	2	3	4	5	6	
Frame	0.025	0.073	0.187	0.370	0.610	0.916	1.260	1:42:43
Truss	0.020	0.041	–	–	–	–	–	1:03:10

VII. Conclusions

In this paper, a stochastic FEM has been proposed to evaluate the response variability of structures whose material property exhibits spatial random variation. The main contribution has been in deriving an analytical or closed form stochastic element stiffness matrix. In consequence, the system equation of any complex truss and frame can be established in a systematic manner. With the aid of the first-order perturbation method, the variability of responses has been determined.

Several remarks are made as follows:

(1) The construction of the stochastic stiffness matrix of a beam element requires division of the element. The division number mainly depends on the correlation distance. The greater the distance is, the smaller the required division number is. The type of AF has some effect, but only when the correlation distance is small.

(2) Although the accuracy of the proposed SFEM has been demonstrated, this accuracy may deteriorate when the random field has higher variance. However, this is primarily due to the first-order approximation rather than to the derived stochastic stiffness matrix.

(3) A numerical study has revealed that response variability can be evaluated by the present method with only minimal computational effort beyond that required for the analysis of a deterministic structure of the same size. Meanwhile, the computational cost is drastically reduced if it is compared with that required in the simulation method. In the past, an ordinary structure could only be solved by using the simulation method.

In conclusion, this paper has provided a highly tractable analysis methodology for determining the response variability of truss and frame structures. Its solution accuracy and computational efficiency has confirmed the feasibility of the present method in dealing with this kind of stochasticity problem.

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材料性質具空間變異的結構分析

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摘　要

因不確定因素而作分析的結構可靠度乃結構設計時的重要參考依據。一個典型的系統隨機性問題即是材料性質具空間上的變異。是以本文旨在推導桁架與梁元素的隨機勁度矩陣，俾傳統的有限元素法得以計算因材料性質具空間隨機變異而引致之結構的反應變異性。

一個結構元素可依隨機場的變動度分爲數個次元素。次元素的彈力特性以隨機場的局部“空間平均”表之。藉靜態濃縮將增加之自由度去除，使新導出的元素勁度矩陣自由度如傳統者。據此以建立系統勁度矩陣與方程式。利用一階擾動法吾人可求得節點位移，其由確定與隨機的兩部分組成。數值例顯示本研究提供一個較易處理的步驟，以求得複雜結構的反應變異性，同時有良好的精度與計算效率。