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Torsional Behavior of High Strength Concrete Beams

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ABSTRACT

This paper describes tests on the torsional behavior of high strength concrete beams. The major variables considered are the longitudinal steel ratio, ρ_l, and the transverse steel ratio, ρ_t. All 19 beams, including 3 beams with no reinforcement, 4 beams with longitudinal steel only, and 12 beams with different longitudinal and transverse steel, were cast. The results revealed that the torsional strength of beams with stirrups at 12 cm spacing (ρ_t=0.01) could be maintained without a drastic drop in torsional strength although the spacing of stirrups was larger than the value suggested by ACI code. A regression equation was derived, based on this study ($f'_{c,ave}$=950 kgf/cm^2), for ultimate torque of reinforced high strength concrete beams under pure torsion.

Key Words: torsion, high strength concrete

I. Introduction

The term high-strength concrete is generally used for concrete with a compressive strength higher than 420 kgf/cm^2 (ACI Committee 363, 1984). Further evaluation of the properties of concrete, such as the brittle behavior of high strength concrete over the 420 kgf/cm^2, is important for the design of torsion. The object of this research was to study the behavior of reinforced high-strength concrete beams under the action of pure torsion and to evaluate the applicability of the provisions suggested by ACI code.

II. Basic Equations

St. Venant (1856) invented a semi-inverse method to solve the torsion problem for noncircular cross sections, and found that the maximum shear stress, τ_{max}, will occur at the midpoint of the wider face for a rectangular section:

$$\tau_{max}=kG\theta x. \quad (1)$$

The torsional moment, T, is:

$$T=\beta x^3 yG\theta. \quad (2)$$

The relationship between T and τ_{max} is as follows:

$$T=\alpha x^2 y\tau_{max}, \quad (3)$$

where
x=shorter dimension of a rectangular cross section;
y=larger dimension of a rectangular cross section;

$$k=\left(1-\frac{8}{\pi^2}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n^2\cosh\frac{n\pi y}{2x}}\right);$$

$$\beta=\frac{1}{3}\left(1-\frac{192}{\pi^5}\frac{x}{y}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n^5}\tanh\frac{n\pi y}{2x}\right);$$

$$\alpha=\frac{\beta}{k},\ \frac{y}{x}=1.0\sim\infty\ \text{ for }\ \alpha=0.208\sim0.333.$$

Bach and Graf (1912) first used elastic theory. The following failure criterion is assumed: Torsional failure of a plain concrete member occurs when the maximum principal tensile stress equals the tensile strength of the concrete, f'_t, that is:

$$T'_e=\alpha x^2 y f'_t, \quad (4)$$

where α=St. Venant's coefficient.

Nylander (1945) surmised that the extra strength could be contributed by the plastic property of concrete. The plastic failure torque, T_p, can be expressed as:

$$T_p = \alpha_p x^2 y f_t' , \tag{5}$$

where

$$\alpha_p = (0.5 - \frac{x}{6y}).$$

Hsu (1968a) presented the skew-bending theory with the results of tests on rectangular plain concrete beams and gave the nominal torsional strength, T_{np}, as follows:

$$T_{np} = \frac{x^2 y}{3} f_r , \tag{6}$$

where f_r=modulus of rupture of concrete.

Also, according to tests by McHenry and Karni (1958), the perpendicular compression will reduce the tensile strength of concrete by 15%, so Eq. (6) is further revised as follows:

$$T_{np} = \frac{x^2 y}{3} 0.85 f_r . \tag{7}$$

Bakhsh *et al.* (1990), through the test results of 13 rectangular plain high-strength concrete beams (400-900 kgf/cm²) subjected to pure torsion, found that the skew-bending theory is suitable for predicting the torsional capacity of plain high-strength concrete beams, provided that the splitting tensile strength, f_{sp}, is taken to represent the concrete tensile strength, i.e.:

$$T_n = \frac{x^2 y}{3} f_{sp} . \tag{8}$$

Rausch (1929) proposed the Space Truss Analogy theory for reinforced concrete subjected to torsion and derived the torsional strength, T_n, as:

$$T_n = \frac{2 x_1 y_1 A_t f_{ty}}{s} , \tag{9}$$

where
x_1 = shorter center-to-center dimension of closed rectangular stirrup;
y_1 = longer center-to-center dimension of closed rectangular stirrup;
A_t = area of one leg of a closed stirrup;
f_{ty} = yield stress of stirrup;
s = spacing of stirrups.

Andersen (1935) suggested that Rausch's equation should be modified by an efficiency coefficient less than unity, and that the concrete should also contribute to the torsional resistance. Andersen's equation is expressed as follows:

$$T_n = T_e + \lambda \frac{2 x_1 y_1 A_t f_{ty}}{s} , \tag{10}$$

where
$T_e = \alpha x^2 y f_t$ =torsional resistance of plain concrete taken as the elastic torque;
λ = efficiency coefficient of reinforcement.

Cowan (1950), using a strain energy method, derived and suggested the efficient coefficient λ=0.8, that is:

$$T_n = T_e + 0.8 \frac{2 x_1 y_1 A_t f_{ty}}{s} = T_e + 1.6 \frac{x_1 y_1 A_t f_{ty}}{s} . \tag{11}$$

Hsu (1968b) proposed a new failure surface with the results of tests. This surface is assumed to be a plane perpendicular to the wider face and inclined at 45° to the axis of the beam, and they then derived the ultimate torque of beam, T_u, having the form:

$$T_u = T' + \Omega \frac{x_1 y_1 A_t f_{ty}}{s} , \tag{12}$$

where
$T' = \frac{1.015}{\sqrt[3]{x}} x^2 y \sqrt{f_c'} \ (kgf-cm)$;
$\Omega = 0.66m + 0.33 \frac{y_1}{x_1}$;
$m = \frac{s A_l}{2 A_t (x_1 + y_1)}$, if $f_{ly} \neq f_{ty}$, multiply m with $\frac{f_{ly}}{f_{ty}}$;
A_l = Section area of total longitudinal steels (cm²);
A_t = Section area of one leg of a stirrup (cm²);
f_{ly} = Yield stress of longitudinal steel (kgf/cm²).
However, this equation is only suitable for the underreinforced condition in which both longitudinal and transverse steel yields before the failure of concrete.

ACI Committee 318 (1971) established the design provisions for reinforced beams subjected to torsion, that is:

$$T_u = 0.212 x^2 y \sqrt{f_c'} + \alpha_t \frac{x_1 y_1 A_t f_{ty}}{s} , \tag{13}$$

where $\alpha_t = 0.66 + 0.33 \frac{y_1}{x_1} \leq 1.50$.

McMullen and Rangan (1978), from the results of torsion tests on ten rectangular reinforced concrete beams with the principal variables being the aspect ratio and the amount of reinforcement, showed that, other things being constant, the strength decreases with an increase in aspect ratio, and that the efficiency coefficient of reinforcement is related to $\sqrt{m'}$:

$$T_u = 0.636\sqrt{f_c'}\, kx^2y + 1.4\sqrt{m'}\, \frac{A_t x_1 y_1 f_{ty}}{s}, \tag{14}$$

where

$$k = \frac{0.5}{1+\frac{x}{y}} \le 0.33\,, \quad m' = \frac{sA_l f_{ly}}{2A_t(x_1+y_1)f_{ty}}\,.$$

For a beam subjected to pure torsion, a minimum amount of torsional reinforcement must be provided such that $T_u = T_{cr}$, and a minimum torsional reinforcement, $A_{t,\,\min}$ (cm^2), is required to ensure the ductility of the beam when it cracks (Hsu, 1984):

$$A_{t,\min} = \frac{xs}{f_{ty}}\left[\left(\frac{0.318}{\alpha_t}\right)\left(\frac{x}{x_1}\right)\left(\frac{y}{y_1}\right)\sqrt{f_c'}\,\right]. \tag{15}$$

III. Test Program

In view of the heterogeneity of concrete, this study was carried out by testing to analyze the behavior of members subjected to pure torsion. The principal parameters considered were the longitudinal steel ratio, ρ_l, and transverse steel ratio, ρ_t. All 19 beams were cast, including 3 beams of plain concrete, 4 beams with longitudinal steel only, and 12 beams with both longitudinal and transverse steel. Details of the beams are shown in Table 1. In the designation of a specimen, the letters A and B represent the same size of longitudinal steel with differing yield stress, N represents beams with no longitudinal steel, the first number is the longitudinal bar size, the second number is the size of stirrups, the third number is the spacing of stirrups, and the last number is the specimen number.

All test specimens were 15×30 cm in section and 230 cm long, with a clear torque span of 90 cm. To prevent failure at the ends of beams, extra reinforcement was added to the two end parts of each beam. The concrete was placed for each beam in one batch using a tilting mixer. All beams and control specimens, comprised of 10×20 cm cylinders and 15×15×50 cm beams, were cast and cured under the same conditions. A general purpose type I portland cement was used. Aggregates were from the Ta-Han river. Coarse aggregates had a maximum size of 1.5 cm. Elkem Microsilica was used. Superplasticizer was used to lower the water content requirement of the concrete mix, provide good workability, and increase the strength by virtue of a low water-cement ratio (w/c). Table 2 shows the details of the mix proportion. Concrete strengths are shown in Table 3. Reinforcement used in this study were from the Shin-Zon steel company. The steel properties are shown in Table 4.

Table 1. Details of Specimens

Series	Specimen code	Longitudinal bar			Stirrups			m'	ρ_{total}
		No.	ρ_l	f_{ly} kgf/cm^2	No.	ρ_t	f_{ty} kgf/cm^2		
1	N-0-0-00-1	---	---	---	---	---	---	---	---
	N-0-0-00-2	---	---	---	---	---	---	---	---
	N-0-0-00-3	---	---	---	---	---	---	---	---
2	A-4-0-00-1	4-#4	0.0107	3905	---	---	---	---	0.0107
	A-4-0-00-2	4-#4	0.0107	3905	---	---	---	---	0.0107
	B-4-0-00-1	4-#4	0.0110	3438	---	---	---	---	0.0101
	B-4-0-00-2	4-#4	0.0110	3438	---	---	---	---	0.0101
3	B-4-3-12-1	4-#4	0.0110	3438	#3@12	0.0101	4071	0.920	0.0211
	B-4-3-12-2	4-#4	0 0110	3438	#3@12	0.0101	4071	0.920	0 0211
4	A-4-3-09-1	4-#4	0.0107	3905	#3@9	0.0134	4071	0.766	0.0241
	A-4-3-09-2	4-#4	0.0107	3905	#3@9	0.0134	4071	0.766	0.0241
	B-4-3-09-1	4-#4	0.0110	3438	#3@9	0.0134	4071	0.693	0.0244
	B-4-3-09-2	4-#4	0.0110	3438	#3@9	0.0134	4071	0.693	0.0244
5	A-5-3-06-1	4-#5	0.0174	3707	#3@6	0.0201	4071	0.788	0.0375
	A-5-3-06-2	4-#5	0.0174	3707	#3@6	0.0201	4071	0.788	0.0375
	B-5-3-06-1	4-#5	0.0173	3603	#3@6	0.0201	4071	0.762	0.0374
	B-5-3-06-2	4-#5	0.0173	3603	#3@6	0.0201	4071	0.762	0.0374
6	A-6-3-04-1	4-#6	0.0257	3800	#3@4	0.0302	4071	0.794	0.0559
	A-6-3-04-2	4-#6	0.0257	3800	#3@4	0.0302	4071	0.794	0.0559

Table 2. Concrete Proportion (unit: kg/m^3)

Coarse aggregate	Fine aggregate	Cement	Silica fume	Superplasticizer	Water
990	660	530	80	12	152

Table 3. Test Results of Concrete Strength

Compressive strength			Splitting strength			Modulus of rupture		
average kgf/cm^2	standard deviation kgf/cm^2	coefficient of varience %	average kgf/cm^2	standard deviation kgf/cm^2	coefficient of variance %	average kgf/cm^2	standard deviation kgf/cm^2	coefficient of variance %
948.8	141.9	14.96	57.74	4.98	8.62	63.95	7.04	11.00

Table 4. Properties of Reinforcement

Bar no.	Diameter (cm)	Area (cm^2)	Yield stress (kgf/cm^2)
#3	0.967	0.734	4071
#4 (A)	1.239	1.206	3905
#4 (B)	1.253	1.233	3438
#5 (A)	1.580	1.961	3707
#5 (B)	1.573	1.943	3603
#6 (A)	1.919	2.892	3800

The beams were tested under the action of pure torsion. A special-design setup was made by a local machine factory and is shown in Fig. 1. Special bearings at the support insured that the test beam was free to twist at one end while the other end was held against torsional rotation. The angle of twist was measured over a 90 cm gauge length. There was a pair of clamps with a cantilever arm which were 90 cm apart and had an arm length 77.5 cm away from the beam axis, as shown in Fig. 2. Then the twist angle per unit length could be obtained by dividing the relative displacement

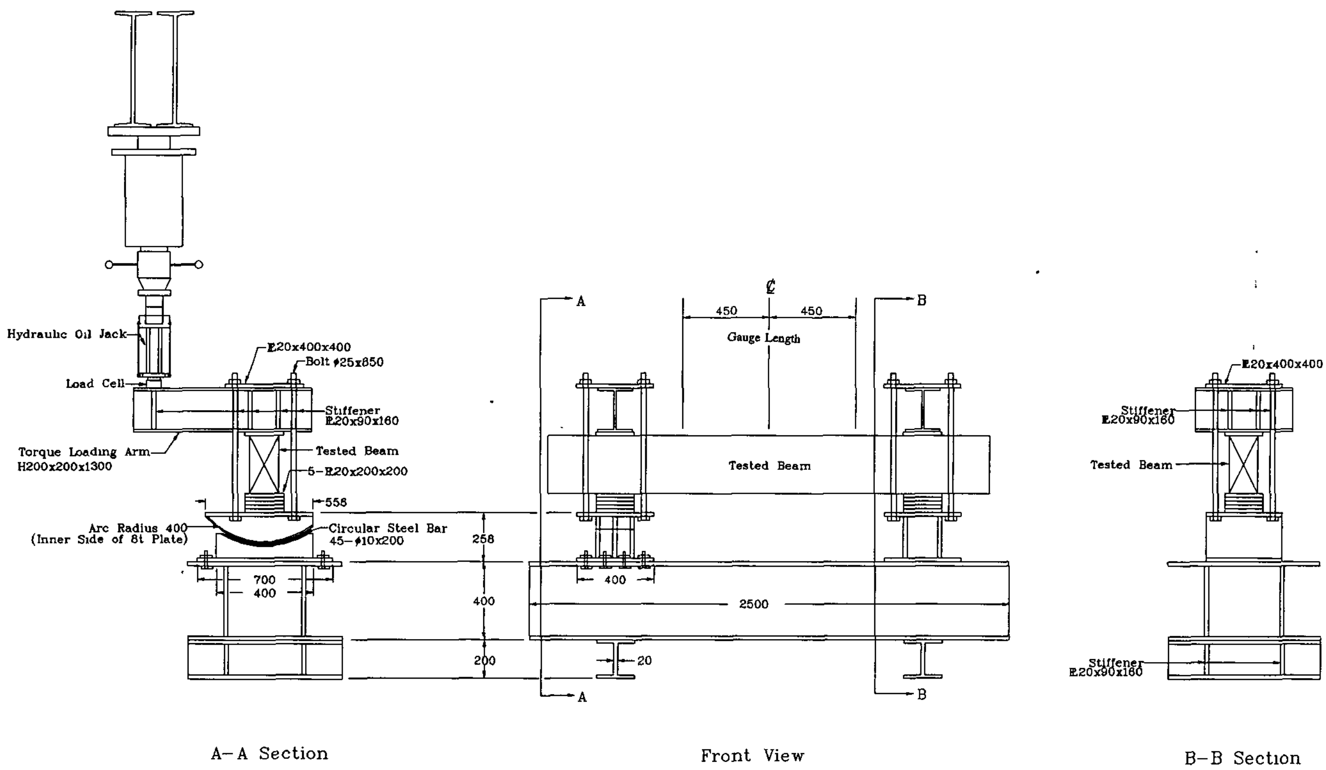

Fig. 1. Torque loading setup.
(Unit: mm)

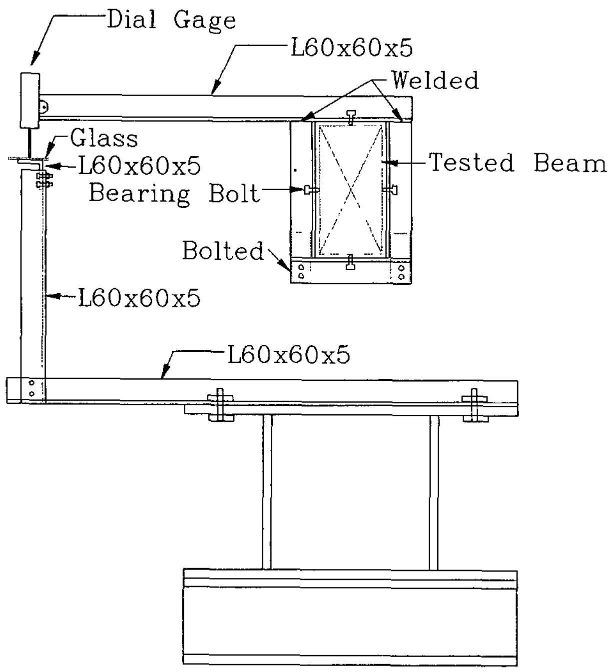

Fig. 2. Twist Measuring Arrangement.

between the ends of two cantilever arms by the gauge length, 90 cm, and cantilever arm length, 77.5 cm. Data reading equipment, including a load cell and dial gauge, were used to record the magnitude of load and displacement during the loading. Data were read at a fixed time interval and load increment.

IV. Test Results

The test data are summarized in Table 5. The behavior of the tested beams under pure torsion is described as follows:

(1) Series 1, specimens N-0-0-00-1, N-0-0-00-2 and N-0-0-00-3, were cast with plain concrete. The T-θ curves were almost straight, and one of these curves is shown in Fig. 3. Failure of beams occurred immediately after cracking initiated.

(2) Series 2, specimens A-4-0-00-1, A-4-0-00-2, B-4-0-00-1 and B-4-0-00-2, only had four #4 longitudinal steel. The torque-twist curve does not deviate excessively from a straight line. The torsional capacity descended vigorously to some residual strength after cracking occurred, shown as Fig. 4. It can be seen from the splitting phenomenon at the beam section corner that the residual strength was obtained from the dowel action of the longitudinal steel.

(3) Series 3, specimens B-4-3-12-1 and B-4-3-12-2, were arranged with 4-#4 longitudinal steel and #3

Table 5. Test Results of Beams

Series	Specimen code	Ultimate torque (kgf-cm)	Cracking torque (kgf-cm)	θ_u (rad/cm×10^{-5})	θ_{cr} (rad/cm×10^{-5})	$(\frac{dT}{d\theta})_o$ (×10^9)	$(\frac{dT}{d\theta})_s$ (×10^9)
1	N-0-0-00-1	92552	92552	3.068	3.068	3.568	3.359
	N-0-0-00-2	86679	86679	9.004	9.004	2.721	2.742
	N-0-0-00-3	106260	106260	3.047	3.047	3.642	3.615
2	A-4-0-00-1	110572	110572	3.355	3.355	3.425	3.415
	A-4-0-00-2	100331	100331	3.111	3.111	3.891	3.615
	B-4-0-00-1	133749	133749	4.330	4.330	3.775	3.468
	B-4-0-00-2	100331	100331	3.470	3.470	3.718	3.464
3	B-4-3-12-1	134673	107184	53.505	*	*	*
	B-4-3-12-2	130977	100947	50.194	2.982	3.876	3.645
4	A-4-3-09-1	138369	95711	65.419	3.455	3.535	3.254
	A-4-3-09-2	148841	120736	51.584	*	*	*
	B-4-3-09-1	154770	108416	60.215	3.412	3.913	3.630
	B-4-3-09-2	163394	101871	68.416	3.283	4.111	3.720
5	A-5-3-06-1	200739	96635	46.681	3.498	3.763	3.596
	A-5-3-06-2	181951	92939	46.093	2.910	3.881	3.578
	B-5-3-06-1	194887	84007	64.932	3.340	3.748	3.291
	B-5-3-06-2	197967	84931	64.229	3.312	4.096	3.712
6	A-6-3-04-1	221760	67298	68.616	2.867	3.274	2.887
	A-6-3-04-2	215908	71918	60.330	2.767	3.625	3.233

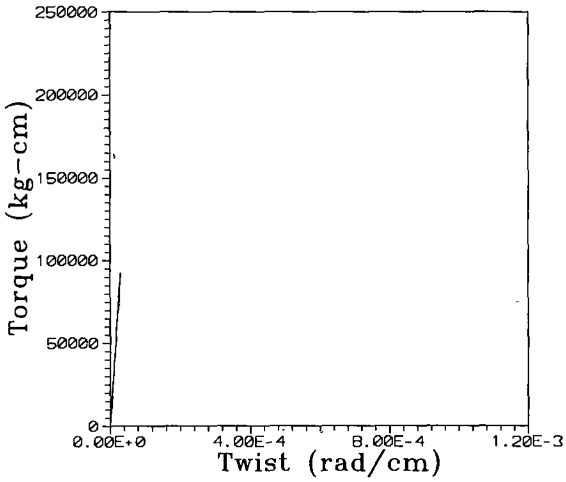

Fig. 3. Torque-twist relationship of A-0-0-00-1.

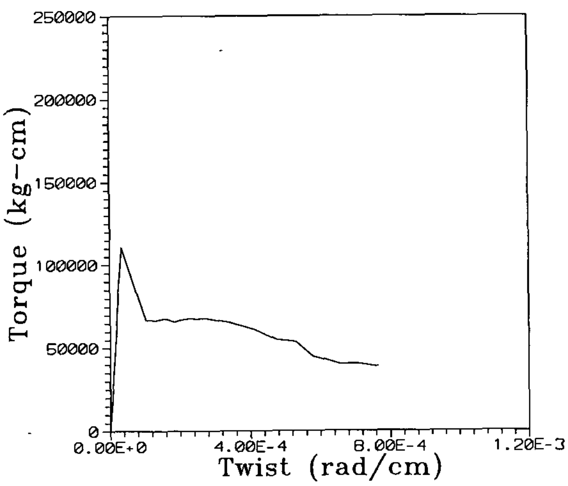

Fig. 4. Torque-twist relationship of A-4-0-00-1.

stirrups with 12 cm spacing. The torsional stiffness of this group of beams was reduced distinctly after cracking. At the same time, the longitudinal bar began to take the loading to maintain the cracking torque of the beam, and after that, there was a continuous increase of twist angle but with little increase of torque capacity while number of added cracks and the width of the cracks increased. The *T-θ* curves have a large plateau as shown in Fig. 5. The beams did not fail until the inclined concrete struts between the cracks on the wider face of the beam fractured.

(4) Series 4, specimens A-4-3-09-1, A-4-3-09-2, B-4-3-09-1 and B-4-3-09-2, with 4-#4 longitudinal bar

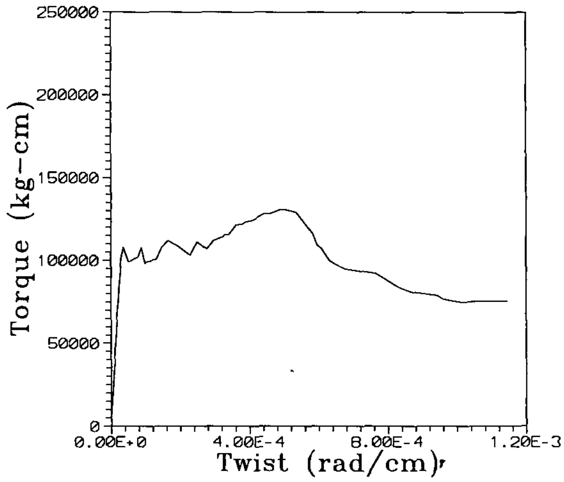

Fig. 5. Torque-twist relationship of B-4-3-12-2.

and #3 sturrups at 9 cm spacing, have *T-θ* curves with a larger plateau and could still resist increasing torque, consistent with a larger twist angle, until failure of beams occurred. One of the curves is shown in Fig. 6.

(5) Series 5, specimens A-5-3-06-1, A-5-3-06-2, B-5-3-06-1 and B-5-3-06-2, had 4-#5 longitudinal steel and #3 stirrups at 6 cm spacing. Specimen A-5-3-06-1 was loaded with cyclic loading (loading and unloading). It can be seen from the *T-θ* curve, shown in Fig. 7, that the torsional stiffness did not decay distinctly if the inclined concrete struts between the cracks did not fracture. Specimen A-5-3-06-2 was subjected to an initial load such that the inclined concrete struts themselves had second-

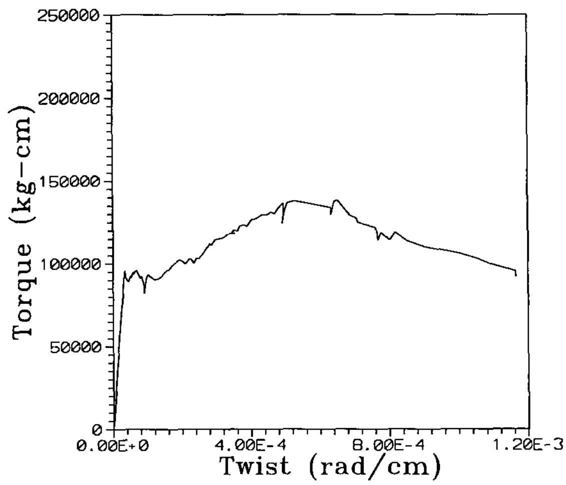

Fig. 6. Torque-twist relationship of A-4-3-09-1.

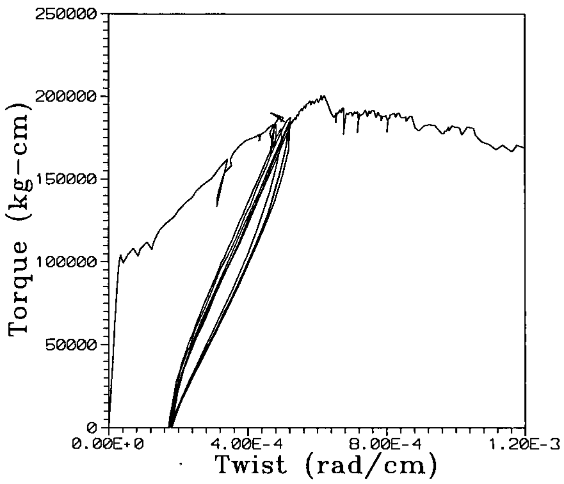

Fig. 7. Torque-twist relationship of A-5-3-06-1.

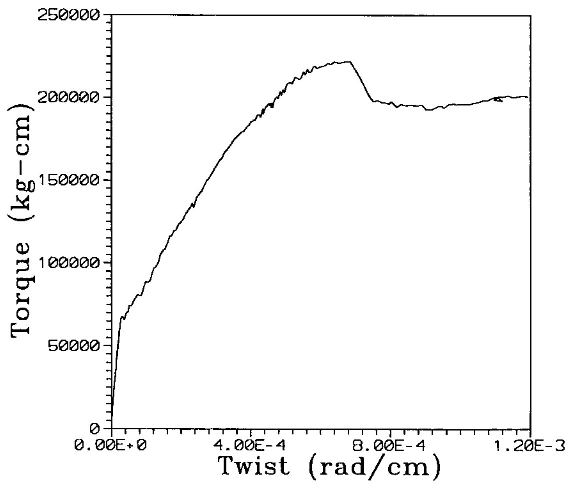

Fig. 9. Torque-twist relationship of A-6-3-04-1.

ary cracks, and was then subjected to cyclic loading. It can be seen from the T-θ curve, shown in Fig. 8, that the torsional stiffness decayed continuously and that the torsional capacity no longer increased.

(6) Series 6, specimens A-6-3-04-1 and A-6-3-04-2, which had 4-#6 longitudinal steel and #3 stirrups at 4 cm spacing, had a much smaller value of cracking torque compared with that for plain concrete beams having the same section. The torsional stiffness did not decrease very strong after cracking and had no distinct plateau, as shown in Fig. 9. The beams did not fail until the inclined concrete struts between cracks fractured.

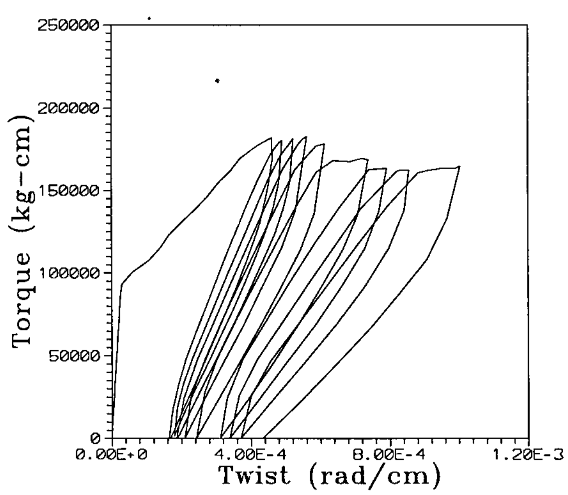

Fig. 8. Torque-twist relationship of A-5-3-06-2.

V. Discussions

From the characteristics of the T-θ curves of beams and related analysis, we can conclude that:

(1) Basically, there were two different types of failure, brittle and ductile. For beams with no reinforcement or longitudinal steel only, brittle failure occurred. Beams with both longitudinal and transverse steel could still resist the external load added after cracking, but with torsional stiffness decay. The saw-tooth shape shown in the T-θ curves is due to the reduction of torsional resistance from the cracking of concrete during torque loading.

(2) The cracking torque for each beam can be obtained from the points of the T-θ curves, at which the slope of torsional stiffness has distinct change.

(3) For reinforced high strength concrete beams subjected to pure torsion, the T-θ curve before cracking is nearly straight. After cracking, the decrease of torsional stiffness depends on the amount of total steel to resist torsion. There is a larger plateau for the beam with less torsional reinforcement. Table 5 shows the initial stiffness and the stiffness value which is the slope of the line connecting the initial point and the point equivalent to half of the cracking torque.

(4) A regression equation for the ultimate torque of reinforced high strength concrete beams under pure torsion was found as the best fit for the test data and is:

$$\frac{T_u}{x^2 y\sqrt{f_c'}} = -1.096\left(\sqrt{m'}\,\frac{A_t x_1 y_1 f_{ty}}{s x^2 y\sqrt{f_c'}}\right)^2$$

$$+2.054\left(\sqrt{m'}\,\frac{A_t x_1 y_1 f_{ty}}{sx^2 y\sqrt{f'_c}}\right)+0.091, \quad \textbf{(16)}$$

and the square of the multiple correlation coefficient, R^2, is 93%. This equation shows that there is a limited ultimate torque capacity with increasing torsional reinforcement due to the limit of concrete strength, as shown in Fig. 10.

(5) Table 6 shows the results of comparing the tested ultimate torque with the calculated value using the formula suggested by ACI and other related formulas. It can be seen that the formula suggested by ACI code, when used to predict the ultimate torsional strength of reinforced high strength concrete beams in this study seems to be unconservative. However, there were better results with the formulas of Hsu and McMullen. From Eq. (16) of this study, T_u is smaller than that of ACI-89 Eq. (11-21). This may be due to the reason that the contribution of concrete in this study is smaller than that of ACI-89 Eq. (11-21). From truss model theory, this may be due to the different softening properties (Hsu and Mo, 1985) of normal and high strength concrete with cracking under shearing.

(6) Using skew-bending theory with modulus of fracture (Hsu, 1968a) to predict the torsional strength of plain high strength concrete beams

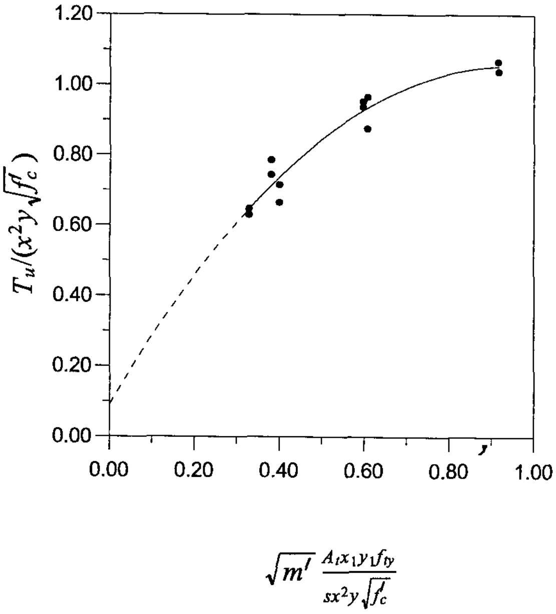

Fig. 10. Ultimate torque vs. steel parameter.

Table 6. Test Results Compared with Reference

Series	Specimen code	$\frac{T_{u,test}}{T_{u,eq.(13)}}$	$\frac{T_{u,test}}{T_{u,eq(12)}}$	$\frac{T_{u,test}}{T_{u,eq(14)}}$	$\frac{T_{u,test}}{T_{u,eq(16)}}$	$\frac{T_{cr,test}}{T_{cr,eq.(7)}}$	$\frac{T_{cr,test}}{T_{cr,eq(8)}}$
1	N-0-0-00-1	---	---	---	---	0.76	0.71
	N-0-0-00-2	---	---	---	---	0.71	0.67
	N-0-0-00-3	---	---	---	---	0.87	0.82
2	A-4-0-00-1	---	---	---	---	0.90	0.85
	A-4-0-00-2	---	---	---	---	0.82	0.77
	B-4-0-00-1	---	---	---	---	1.09	1.03
	B-4-0-00-2	---	---	---	---	0.82	0.77
3	B-4-3-12-1	0.92	0.88	0.96	1.00	0.88	0.83
	B-4-3-12-2	0.89	0.85	0.94	0.97	0.83	0.78
4	A-4-3-09-1	0.77	0.78	0.86	0.90	0.78	0.74
	A-4-3-09-2	0.82	0.84	0.93	0.97	0.99	0.93
	B-4-3-09-1	0.86	0.90	1.00	1.04	0.89	0.83
	B-4-3-09-2	0.90	0.95	1.06	1.10	0.83	0.78
5	A-5-3-06-1	0.81	0.84	0.91	1.03	0.79	0.74
	A-5-3-06-2	0.73	0.76	0.82	0.94	0.76	0.72
	B-5-3-06-1	0.78	0.82	0.89	1.01	0.69	0.65
	B-5-3-06-2	0.79	0.83	0.91	1.03	0.69	0.65
6	A-6-3-04-1	0.63	0.67	0.71	1.01	0.55	0.52
	A-6-3-04-2	0.61	0.65	0.70	0.99	0.59	0.55
Average		0.79	0.81	0.89	1.00	0.80	0.75

seems to give better results than with splitting tensile strength (Bakhsh *et al.*, 1990), but both are still unconservative, as shown in Table 6.

(7) This study has shown that all the beams could still maintain cracking torque after the beams cracked. The $\rho_{total}(=\rho_l+\rho_t)$ varied from 0.0211 to 0.0559 while m' was 0.693~0.920. Furthermore, T_u/T_{cr} was 1.28~3.16, and T_{cr} was 100000~70000 kgf-cm.

(8) Specimens B-4-3-12-1 and B-4-3-12-2 with stirrups at 12 cm spacing, where the spacing was larger than the value ($\frac{x_1+y_1}{4}$ =9cm) suggested by ACI code to avoid the decrease of torsional strength, could still maintain the torsional capacity.

(9) As shown in Fig. 5, the cracking torque was maintained until the failure of the specimen. This means that A_t=0.734 cm^2 used in series 3, which is larger than the minimum reinforcement 0.479 cm^2 calculated by Eq. (15) for normal strength concrete, is sufficient to ensure the ductility of the beam.

VI. Conclusions

From the test results and discussion above, where ρ_{total} was in the range of 0.0211~0.0559 and m' was 0.693~0.920, we can conclude that:

(1) The torsional strength of beams with transverse steel ratio ρ_t=0.01 can be maintained at a value not less than the cracking torque even if the spacing of stirrups is over the limit value suggested by ACI code.

(2) A regression formula derived as Eq. (16), based on this study ($f'_{c,ave}$ =950 kgf/cm^2), can be used to estimate the ultimate torque of reinforced high strength concrete beams.

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References

ACI Committee 318 (1971) Building Code Requirements for Reinforced Concrete (ACI 318-71). pp. 78. American Concrete Institute, Detroit.

ACI Committee 363 (1984) State-of-the-Art Report on High Strength Concrete. pp. 48. American Concrete Institute, Detroit.

ACI Committee 318 (1989) Building Code Requirements for Reinforced Concrete (ACI 318-89). American Concrete Institute, Detroit.

Andersen, P. (1935) Experiments with concrete in torsion. *Transactions, ASCE*, **100**, 949-983.

Bach, B. and O. Graf (1912) *Versuche über die Widerstands Fähigkeit von Beton und Eisenbeton gegen Verdrehung*, Heft 16. Deutscher Ausschuss für Eisenbeton, Wilhelm Ernst, Berlin.

Bakhsh, A. H., F.F. Wafa, and A.A. Akhtaruzzaman, (1990) Torsional behavior of plain high-strength concrete beams. *ACI Structural Journal*, **87**(5), 583-588.

Cowan, H. J. (1950) Elastic theory for torsional strength of rectangular reinforced concrete beams. *Magazine of Concrete Research (London)*, **2**(4), 3-8.

Hsu, T. T. C. (1968a) Torsion of Structural Concrete-Plain Concrete Rectangular Sections. Torsion of Structural Concrete, SP-18, pp. 203-238. American Concrete Institute, Detroit.

Hsu, T. T. C. (1968b) Ultimate torque of reinforced rectangular beams. *Journal of the Structural Division, ASCE*, **94**, ST 2, 485-510.

Hsu, T. T. C. (1984) *Torsion of Reinforced Concrete*, pp. 516. Van Nostrand Reinhold Company, Inc., New York.

Hsu, T. T. C. and Y. L. Mo (1985) Softening of concrete in torsional members - theory and tests. *Journal of the American Concrete Institute, Proc.*, **82**, 290-303.

McHenry, D. and J. Karni (1958) Strength of concrete under combined tensile and compressive stresses. *Journal of the American Concrete Institute, Proc.*, **54**, 829-839.

McMullen, A. E. and V. Rangan (1978) Pure torsion in rectangular sections-A re-examination. *ACI Journal*, **75**, 511-519.

Nylander, H. (1945) Vridning och vridningsinspanning vid betong konstruktiener. Statens Kommittee for Byggnadsforskning, Stockholm, Bullentin No. 3.

Prandtl, L. (1903) Zur torsion von prismatischen staben. *Physik Zeitschrift*, **4**, 758.

Rausch, E. (1929) Design of Reinforced Concrete in Torsion. Ph.D. Thesis, Technische Hochschule, Berlin.

Saint-Venant, B. de (1856) Memoire sur la torsion des prismes. Memoires des savants etrangers, Paris, V. 14, pp. 233-560.

混凝土樑之抗扭力行爲研究

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摘　要

本研究以試驗方式探討高強度混凝土樑的抗扭力行爲。主要變數爲縱向鋼筋比ρ_1及橫向箍筋比ρ_t，共製作15cm×30cm×230cm試體樑19支，其中包含純混凝土樑3支，僅含縱向鋼筋樑4支，其餘12支樑分別配置不等量之縱向筋與橫向筋。試驗結果顯示，本文中橫向箍筋間距12cm（ρ_t=0.01）者，其箍筋間距離大於ACI規範建議值，但在樑開裂後，仍能維持樑之扭矩強度。從本文試驗結果（$f'_{c,ave}$=950 kgf/cm^2）得到一迴歸式，以估計高強度混凝土樑之極限抗扭強度。