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Epicyclic Gear Mechanisms for Multi-Speed Automotive Automatic Transmissions

CHENG-HO HSU, YO-CHUN YEH, AND ZHI-REN YANG

Department of Mechanical Engineering
National Sun Yat-Sen University
Kaohsiung, Taiwan, R.O.C.

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ABSTRACT

Epicyclic gear trains for multi-speed automotive automatic transmissions are rotationally non-isomorphic and contain at least four coaxial links, at most three planet gears, no embedded structure, no redundant planet gears, and no floating carriers. This purpose of this paper is to present an efficient methodology for systematic enumeration of multi-speed epicyclic gear mechanisms. First, an approach is proposed to enumerate all the admissible acyclic graphs. Next, the acyclic graph method is applied to enumerate all the displacement graphs of epicyclic gear mechanisms for each admissible acyclic graph. Then, a computer program which has been successfully developed for automatic enumeration of the displacement graphs of epicyclic gear mechanisms is presented. Finally, we present an atlas of promising epicyclic gear mechanisms with four, five and six coaxial links. The results of this study will provide a useful database for the creation of new multi-speed automotive automatic transmissions.

Key Words: automatic transmission, epicyclic gear mechanism, graph theory

I. Introduction

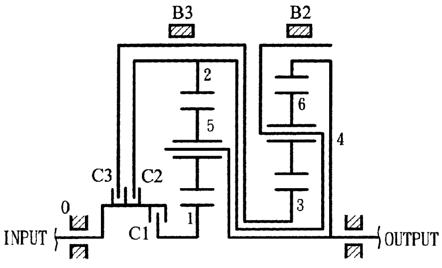
Epicyclic gear trains (EGTs) used in automotive automatic transmissions are called epicyclic gear mechanisms (EGMs) (Chatterjee and Tsai, 1994; Hsieh and Tsai, 1996a, 1996b; Hsu, 1999). Most automotive automatic transmissions provide four forward speed ratios and one reverse speed ratio (Chatterjee and Tsai, 1994; Hsieh and Tsai, 1996a). Chatterjee and Tsai (1994) identified some of the structural characteristics of EGMs and formulated a systematic methodology for enumeration of the graphs of EGMs with up to ten links. Hsieh and Tsai (1996a) showed that seven- and eight-link EGMs having four coaxial links are the optimum mechanisms for three- and four-speed automatic transmissions. Hsu (1999) presented the acyclic graph method for automatic enumeration of displacement graphs of EGMs, and constructed an atlas of EGMs with up to ten links. However, some EGMs having floating carriers result in serious dynamic problems, and some EGMs having four coaxial links are not suitable for five- and six-speed automatic transmissions. Ten-link EGMs, which contain more than three planet gears, are not promising for the design of multi-speed automotive automatic transmissions (Hsieh and Tsai, 1996b), so these EGMs should be further deleted from the atlas presented by Hsu (1999). Moreover, some of them are rotationally isomorphic and have the same kinematic characteristics (Freudenstein, 1971; Hsu, 1994). Therefore, the purpose of this study was to develop an efficient methodology such that rotationally non-isomorphic, promising EGMs for multi-speed automotive automatic transmissions can be enumerated.

In what follows, the fundamental rules of the displacement graphs of multi-speed promising EGMs will first be investigated, and then our methodology for systematic enumeration of displacement graphs of multi-speed promising EGMs will be introduced.

II. Epicyclic Gear Mechanisms

Chatterjee and Tsai (1994) and Hsu (1999) concluded that EGMs should satisfy the following three basic characteristics: (1) an EGM should have at least four coaxial links adjacent to the gearbox, (2) an EGM contains no embedded structure, and (3) an EGM does not contain any redundant links. However, some EGMs having floating carriers result in serious dynamic problems, and some EGMs having more than three planet gears are not practical for five- and six-speed automatic transmissions. Moreover, some EGMs are rotationally isomorphic and have the same kinematic characteristics. Thus, three additional basic characteristics of EGMs, (1) that an EGM does not contain any floating carriers, (2) that an EGM has at most three planet gears, and (3) that EGMs are rotationally non-isomorphic, are presented in this study to make the final result more practical. EGTs satisfying all the basic characteristics are called promising EGMs.

For example, Fig. 1(a) shows a schematic drawing of an EGM used in the GM Hydra-Matic 700-R4 four-speed



(a)

SPEED	C1	C2	C3	B2	B3
FIRST	X			X	
SECOND	X				X
THIRD	X	X			
FOURTH		X			X
REVERSE			X	X	

(b)

Fig. 1. Epicyclic gear mechanism for a four-speed automatic transmission.

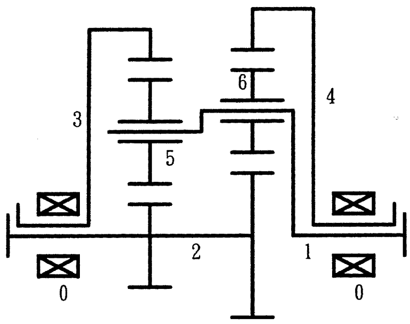
automatic transmission. It is a two-degree-of-freedom, seven-link EGM, which has four coaxial links, 1, 2, 3, and 4, incident to the casing 0 of the gearbox, and two planet gears, 5 and 6, and does not contain any embedded structures, floating carriers or redundant planet gears. Four forward and one reverse speed ratios are obtained through clutching different links to clutches C1, C3 and C4, and brakes B1 and B4. Figure 2(a) and (b) show two different seven-link EGMs, which correspond to the same rotational displacement equations, so they are linearly non-isomorphic and rotationally isomorphic.

III. Graph Representation of Epicyclic Gear Mechanisms

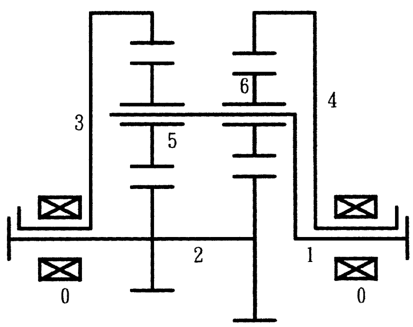
The kinematic structure of an EGM can be represented by its corresponding displacement graph (Hsu, 1999), in which links, simple joints, and multiple joints are denoted by vertices, edges, and solid polygons, respectively. An edge connection between two vertices corresponds to a pair connection between two links, where dashed edges represent gear pairs and solid edges represent revolute pairs. Vertices connected by a common solid polygon in the graph represent coaxial links connected by a multiple joint in the EGM. The vertex representing the gearbox is indicated by a hollow circle and is called the ground-level vertex. The vertices connected to the ground-level vertex by a solid polygon are referred to as first-level vertices, a vertex that is connected to the first-level vertex by one solid edge/polygon is called a second-level vertex and so on. A fundamental circuit (f-circuit), $(i,j)k$, which represents two mating gears i and j, and the associated carrier k, consists of three vertices, one geared edge and two solid edges/polygons. The acyclic graph of an EGM is obtained by removing all the geared edges from its displacement graph. Thus, a displacement graph is a set consisting of an acyclic graph and a subgraph of geared-edges.

For example, the EGM shown in Fig. 1(a) is represented by the displacement graph shown in Fig. 3(a), which has seven vertices, four geared edges, and four f-circuits (1,5)4, (2,5)4, (3,6)2 and (4,6)2. The displacement graph shown in Fig. 3(a) can be decomposed into an acyclic graph as shown in Fig. 3(b) and a subgraph of four geared-edges, (1,5), (2,5), (3,6) and (4,6), as shown in Fig. 3(c). In the displacement (acyclic) graph, vertex 0 is the ground-level vertex, vertices 1, 2, 3 and 4 are the first-level vertices, and vertices 5 and 6 are the second-level vertices. Thus, the displacement graph and acyclic graph shown in Fig. 3(a) and (b) are two-level graphs.

The displacement graph and the rotation graph of an EGM are used to derive the corresponding linear and rotational displacement equations, respectively. The rotation graph of an EGM can be derived from the displacement graph of this EGM (Hsu, 1999). If the rotation graphs of non-isomorphic displacement graphs are identical, then they are said to be linearly non-isomorphic and rotationally isomorphic. It can be shown that displacement graphs having coaxial second-level vertices are rotationally isomorphic to one displacement graph without any coaxial second-level vertices. For example, Fig. 4(a) and (b) show the displacement graphs of the EGMs shown in Fig. 2(a) and (b). Since their rotation graphs are



(a)



(b)

Fig. 2. Rotationally isomorphic epicyclic gear mechanism.

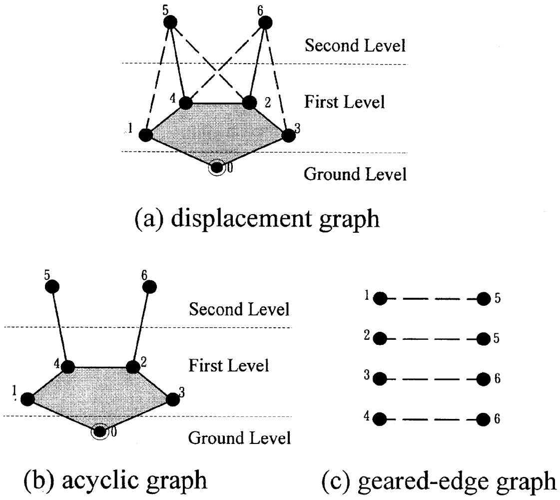


Fig. 3. Graph representation of the EGM shown in Fig. 1(a).

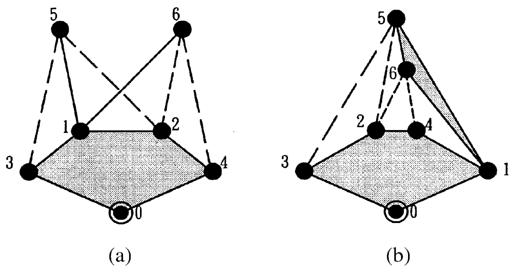


Fig. 4. Rotationally isomorphic displacement graphs.

the same as those shown in Fig. 2(a), they are linear non-isomorphic and rotationally isomorphic. The corresponding acyclic graphs of these EGMs are shown in Fig. 5(a) and (b), respectively, and are called rotationally isomorphic acyclic graphs. It can also be shown that acyclic graphs having coaxial second-level vertices are rotationally isomorphic to one acyclic graph without any coaxial second-level vertices.

For an N-vertex displacement (acyclic) graph, Hsu (1999) also defined the corresponding vertex-vertex adjacency matrix as representing the kinematic structure of EGMs. The elements d_{ij} (a_{ij}) of the corresponding adjacency matrix are defined as follows. If vertex i is adjacent to vertex j with a revolute edge, then $d_{ij} = 1$ ($a_{ij} = 1$). If vertex i is adjacent to vertex j with a geared edge, then $d_{ij} = 2$. If vertex i is adjacent to vertex j with a solid polygon of m vertices, then $d_{ij} = m$ ($a_{ij} = m$); otherwise, $d_{ij} = 0$ ($a_{ij} = 0$). Furthermore, $d_{ii} = 0$ ($a_{ii} = 0$). The adjacency matrix of the (N–3) geared-edges graph of an N-vertex displacement graph is defined as a symmetric matrix of order N with the elements $g_{ij} = 2$ if vertex i is adjacent to vertex j with a geared edge; otherwise, $g_{ij} = 0$. Furthermore, $g_{ii} = 0$.

Since an N-vertex displacement graph can be decomposed into an N-vertex acyclic graph and an (N–3) geared-edge subgraph, the adjacency matrix of the displacement graph $\{d_{ij}\}$ can be expressed as the sum of the adjacency matrices of the acyclic graph $\{a_{ij}\}$ and the geared-edges subgraph $\{g_{ij}\}$, that is,

$$\{d_{ij}\} = \{a_{ij}\} + \{g_{ij}\}. \qquad \textbf{(1)}$$

Using the decomposition algorithm, Hsu (1999) developed a computer program for automatic enumeration of adjacency matrices of displacement graphs of N-link EGTs.

IV. Fundamental Rules for the Displacement Graphs of Epicyclic Gear Mechanisms

Based on the basic characteristics of promising EGMs described in the previous sections, the fundamental rules for the displacement graphs of N-link promising EGMs can be summarized as follows:

F1. The graph contains N vertices, N–3 geared edges, and one pendant vertex as the ground-level vertex.
F2. The graph is a two-level graph, in which the first-level vertices are connected to the ground-level vertex by one solid polygon, and the second-level vertices are connected to the first-level vertex by one edge/polygon.
F3. The graph has at least four first-level vertices and at most three second-level vertices.
F4. The graphs of different EGMs are rotationally non-isomorphic.
F5. The subgraph obtained by removing all the geared edges of the graph is an N-vertex, two-level acyclic graph.
F6. Adding a geared edge to the acyclic graph forms an f-circuit. The graph has N–3 f-circuits.
F7. The graph does not contain any embedded structures.
F8. The graph does not contain any vertices representing redundant gears.

According to the fundamental rules, the systematic enumeration methodology for the displacement graphs of N-link promising EGMs is divided into two stages: the first stage enumerates the catalog of N-vertex admissible acyclic graphs, and the second stage enumerates the displacement graphs of all the N-link EGMs.

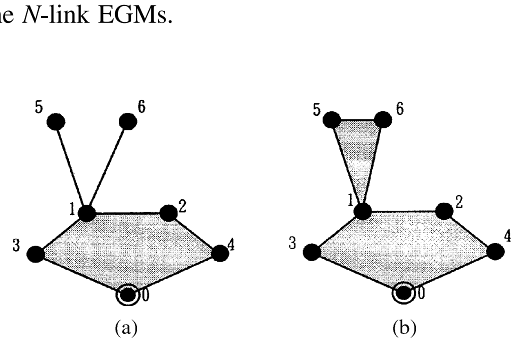


Fig. 5. Acyclic graphs of rotationally isomorphic displacement graphs.

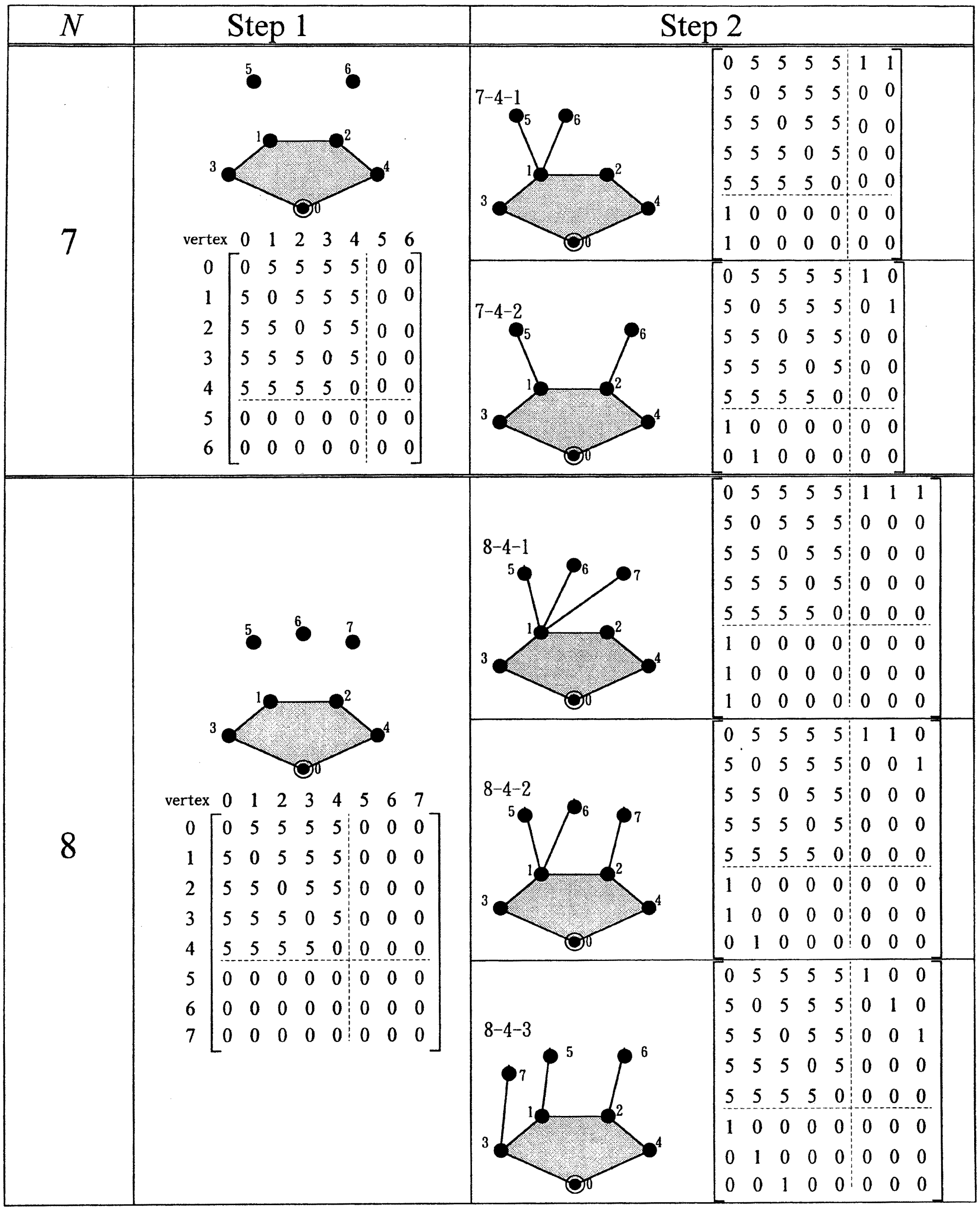


Fig. 6. The enumeration process for seven- and eight-vertex admissible acyclic graphs having four first-level vertices.

V. Admissible Acyclic Graphs

According to fundamental rules F1 to F5, the admissible acyclic graphs of N-link promising EGMs are two-level graphs consisting of one ground-level vertex, k first-level vertices, and N–k–1 second-level vertices. Moreover, acyclic graphs having second-level coaxial vertices are rotationally isomorphic to one acyclic graph without any second-level coaxial vertices. Based on the above facts, the procedure to enumerate the admissible acyclic graphs of N-link EGMs having k coaxial links can be summarized as follows.

Step 1. Draw a solid polygon of k+1 vertices, consisting of one ground-level vertex and k first-level vertices.

Step 2. Connect the N–k–1 second-level vertices to the first-level vertices to generate all the non-isomorphic N-vertex admissible acyclic graphs.

Using the notation of adjacency matrices, a computer program has been developed for automatic enumeration of the adjacency matrices of admissible acyclic graphs for N-link promising EGMs. Figure 6 shows the enumeration process for seven- and eight-vertex admissible acyclic graphs having four first-level coaxial vertices. Figures 7 and 8 show the atlases of the admissible acyclic graphs having five and six first-level coaxial vertices, respectively.

VI. Enumeration of Epicyclic Gear Mechanisms

It is obvious that N-vertex geared graphs can be synthesized from N-vertex admissible acyclic graphs by adding N–3 geared edges to form N–3 f-circuits. However, some synthesized geared graphs are open graphs, some contain embedded structure, some are structurally isomorphic, and some contain redundant links. Such graphs violate the fundamental rules and should be weeded out. Moreover, displacement graphs synthesized from different acyclic graphs are non-isomorphic, so N-vertex displacement graphs can be individually synthesized from each N-vertex admissible acyclic graph. Based on these facts, the systematic procedure for enumeration of N-vertex displacement graphs from each admissible acyclic graph can be summarized as follows.

Step 1. Enumerate all the possible geared graphs.

All the possible N-vertex geared graphs are enumerated from an N-vertex admissible acyclic graph in the following steps: (1) identify all the K possible f-circuits which can be formed in the acyclic graph, and (2) select any (N-3) f-circuits from K possible f-circuits to form a geared graph; then $C(K, N–3)$ geared graphs are generated.

Step 2. Identify and delete open geared graphs.

Some enumerated geared graphs, which contain at least one pendant first-level vertex, are open graphs. They are identified and are deleted from the set of enumerated geared graphs.

Step 3. Identify and delete degenerate geared graphs.

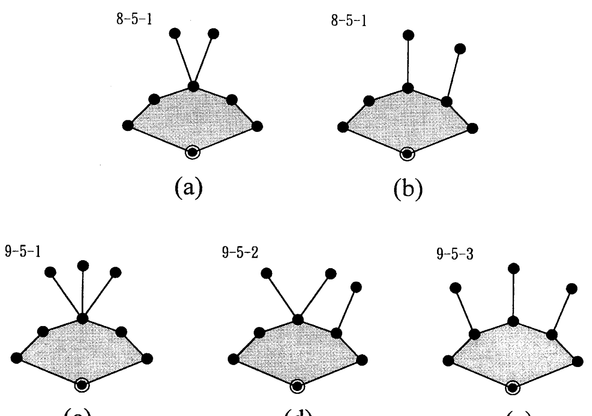


Fig. 7. Admissible acyclic graphs having five first-level vertices.

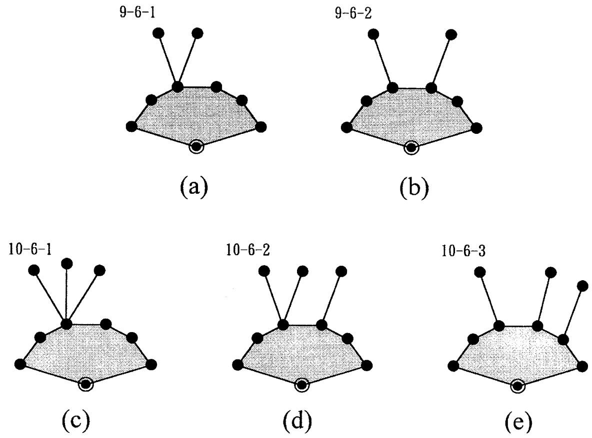


Fig. 8. Admissible acyclic graphs having six first-level vertices.

An m-link gear structure is a zero-degree-of-freedom geared chain and consists of m links, m–1 revolute pairs and m–1 gear pairs. The graph of an m-link gear structure is an m-vertex geared graph which has m–1 f-circuits. Geared graphs containing any embedded gear structure are called degenerate geared graphs and should be deleted. Based on these facts, Hsu and Wu (1997) developed a computer program to detect whether a geared graph is a degenerate geared graph.

Step 4. Detect structural isomorphism of geared graphs by comparing their structural codes.

Isomorphic geared graphs are identified by comparing their structural codes (Hsu, 1994), and all the non-isomorphic displacement graphs of EGTs are synthesized.

Step 5. Identify and delete displacement graphs having redundant planet gears to obtain all the displacement graphs of promising EGMs.

A redundant planet gear in an EGM is a planet gear adjacent to only one mating gear and its carrier. A computer program has been developed (Hsu and Wu, 1997) that automatically detects any redundant planet gears in an EGT. The developed computer program can be used to detect whether or not geared graphs enumerated in Step 4 have any redundant planet gears.

Computer subprograms that detect open geared graphs (Hsu, 1999) and degenerate geared graphs (Hsu and Wu, 1997), identify isomorphic geared graphs (Hsu, 1994) and check for the existence of redundant planet gears (Hsu and Lin, 1994) have been developed. Based on these subprograms, a computer program has been developed for automatic enumeration of N-vertex displacement graphs from each admissible acyclic graph.

VII. Results

Using the proposed methodology, all the admissible

Table 1. Enumeration of Promising EGMs Having Four, Five, and Six Coaxial Links

Acyclic graph	Step1	Step2	Step3	Step4	Step5
7-4-1	35	20	20	4	2
7-4-2	15	13	13	5	3
8-4-1	726	378	324	16	3
8-4-2	252	179	145	42	4
8-4-3	126	102	90	18	0
8-5-1	126	48	48	5	3
8-5-2	56	38	38	6	4
9-5-1	5005	1641	1323	26	9
9-5-2	1716	994	756	90	25
9-5-3	972	679	550	56	16
9-6-1	462	112	112	6	4
9-6-2	210	104	104	9	7
10-6-1	31824	6729	5184	37	15
10-6-2	11440	4924	3591	158	65
10-6-3	6435	3870	2970	120	54

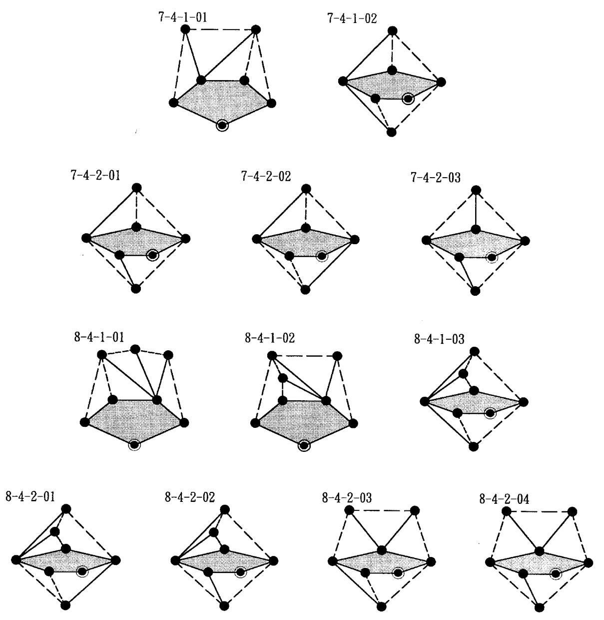


Fig. 9. Displacement graphs of EGMs having four coaxial links.

acyclic graphs of promising EGMs having four, five, and six coaxial links were first enumerated as shown in Figs. 6, 7 and 8. For each admissible acyclic graph, all the displacement graphs of promising EGMs were then enumerated. Using the developed computer program, displacement graphs of promising EGMs were automatically enumerated. Table 1 lists the number of EGTs and promising EGMs having four, five, and six coaxial links synthesized from the admissible acyclic graphs shown in Figs. 6, 7 and 8. Figure 9 shows the atlas of the displacement graphs of 12 promising EGMs having four coaxial links. Figure 10 shows the atlas of the displacement graphs of 57 promising EGMs having five coaxial links. Figure 11 shows the atlas of the displacement graphs of 11 nine-link promising EGMs having six coaxial links. Moreover, we have constructed an atlas for the displacement graph of 134 ten-link promising EGMs having six coaxial links.

VIII. Conclusion

In this study, three important basic characteristics of

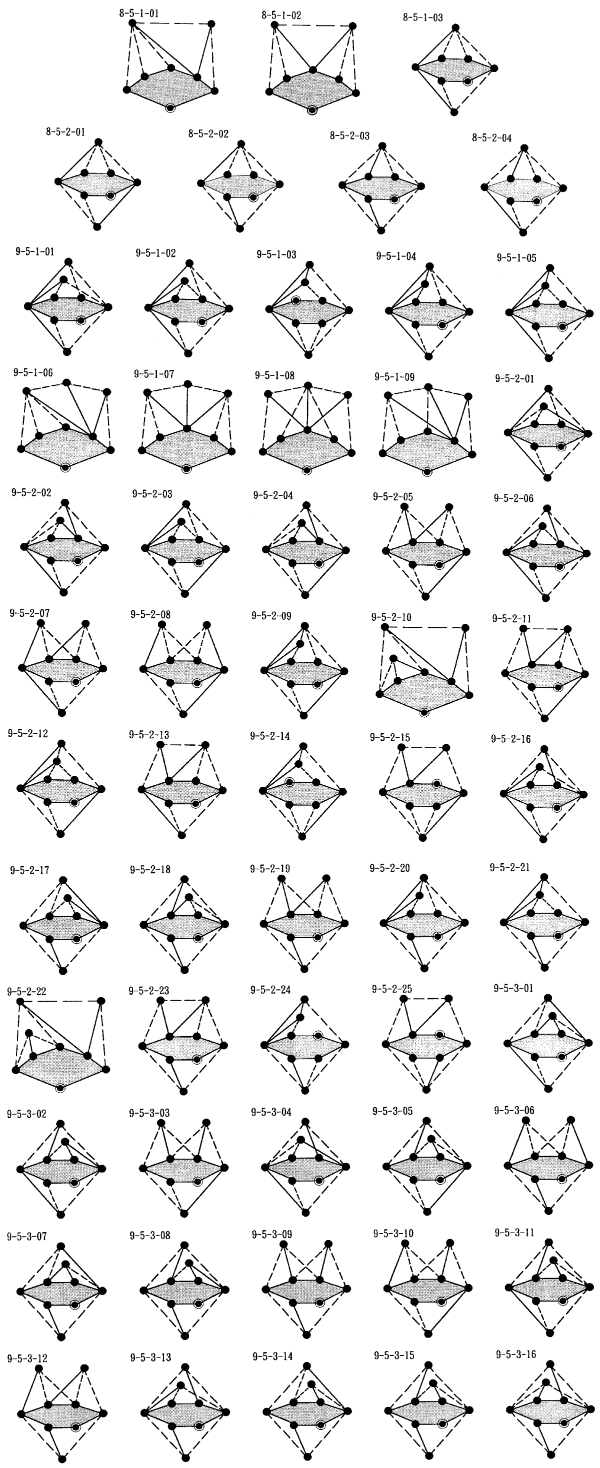


Fig. 10. Displacement graphs of EGMs having five coaxial links.

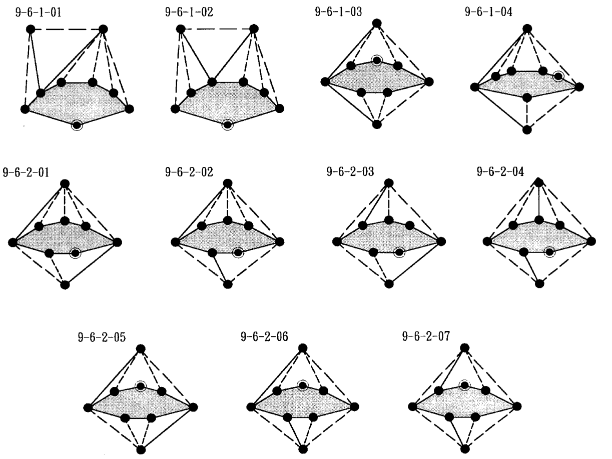


Fig. 11. Displacement graphs of nine-links EGMs having six coaxial links.

enumerated promising EGMs for multi-speed automatic transmissions have been introduced. An efficient methodology has been developed for systematic enumeration of promising EGMs for automatic transmissions. An atlas of displacement graphs of promising EGMs having four, five, and six coaxial links has been constructed. This methodology can also be applied to enumerate EGMs having more than six coaxial links. It is hoped that this work will provide a basis for the development of novel multi-speed automatic transmissions.

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References

Chatterjee, G. and L. W. Tsai (1994) *Enumeration of Epicyclic-type Automatic Transmission Gear Trains.* Transactions of SAE Technical Paper. 941012, SAE, Warrendale, PA, U.S.A.

Freudenstein, F. (1971) An application of Boolean algebra to the motion of epicyclic drives. *ASME Journal of Engineering for Industry*, **93B**, 176-182.

Hsieh, H. I. and L. W. Tsai (1996a) *A Methodology for Enumeration of Clutching Sequences Associated with Epicyclic-type Automatic Transmission Mechanisms.* Transactions of SAE Technical Paper 960719, SAE, Warrendale, PA, U.S.A.

Hsieh, H. I. and L. W. Tsai (1996b) Kinematic analysis of epicyclic-type transmission mechanisms using the concept of fundamental geared entities. *ASME Journal of Mechanical Design*, **118**, 294-299.

Hsu, C. H. (1994) Displacement isomorphism of planetary gear trains. *Mechanism and Machine Theory*, **29**, 513-523.

Hsu, C. H. (1999) Systematic enumeration of epicyclic gear mechanisms for automotives. *JSME International Journal*, Series C, **42**, 225-233.

Hsu, C. H. and Y. L. Lin (1994) Automatic identification of redundant links in planetary gear trains. *Mathematical and Computer Modeling*, **19**, 67-81.

Hsu, C. H. and Y. C. Wu (1997) Automatic detection of embedded structure in planetary gear trains. *ASME Journal of Mechanical Design*, **119**, 315-318.

多速車輛自動變速器之行星齒輪機構

許正和　葉祐君　楊智仁

國立中山大學機械工程學系

摘　要

多速車輛自動變速器之行星齒輪機構具有旋轉不同構的構造，至少含有四個共軸構件，不含有內藏結構體、贅餘行星齒輪及浮動行星臂等特性。本文之目的在於提出一種有效的簡單方法，做爲行星齒輪機構系統化合成的依據。文中首先提出列出可用非循環圖畫的可行方法；其次提出合成齒輪機構的系統化程序，接著以鄰接矩陣爲基礎提出一可行的電腦演算法則，並發展電腦程式；最後以所發展的電腦程式合成含有四至六個共軸構件的行星齒輪機構目錄。本文所得之結果將有利於開發新型多速車輛自動變速器。