Proc. Natl. Sci. Counc. ROC(A)
Vol. 23, No. 2, 1999. pp. 169-180

(Invited Review Paper)

Studies of the Frequency-Magnitude Relation of Earthquakes Based on a One-Dimensional Dynamical Lattice Model

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(Received October 14, 1997; Accepted July 30, 1998)

ABSTRACT

Based on the 1-D dynamical lattice model proposed by R. Burridge and L. Knopoff in their 1967's paper, with velocity-dependent friction and a uniform or an inhomogeneous distribution of the breaking strengths (i.e., static friction strength), the Gutenberg-Richter-type frequency-magnitude (FM) relation has been studied by numerous authors. In this work, the publications on the effects on the FM relation and its scaling exponent, i.e., the *b*-value, of earthquakes due to model parameters are reviewed. The main model parameters include the decreasing rate, *r*, of the dynamic frictional force with sliding velocity, the degree of heterogeneity of the distribution of the breaking strengths, the stiffness ratio *s*, defined as the ratio of the stiffness of the coil spring between two mass elements to that of the leaf spring between a mass element and the moving plate, the friction drop ratio, *g*, of the minimum dynamic frictional force to the breaking strength and the maximum breaking strength, F_{omax}. Some authors have used a fractal distribution of the breaking strengths. The fractal dimension is used to define such a distribution.

The main simulation results show that three kinds of model events are generated. They are microscopic, localized, and de-localized events. Localized events exhibit the Gutenberg-Richter-type FM relation, but this is not the case for the other two kinds of events. The range of magnitudes of localized events depends upon the stiffness ratio *s*. The FM relation and the *b*-value are remarkably affected by the type of friction law, the weakening rate, *r*, the friction drop ratio, *g*, and the maximum breaking strength, F_{omax}, but not by the fractal dimension, *D*, of the distribution of the breaking strengths. The *b*-value of the cumulative frequency-magnitude relation is less than that of the discrete frequency-magnitude relation. There exists a power-law relation between *b* and *s*: $b{\sim}s^{-2/3}$ for the cumulative frequency-magnitude relation and $b{\sim}s^{-1/2}$ for the discrete frequency-magnitude relation. Such a power-law relation does not depend upon *r*, *g*, and F_{omax}.

Key Words: the Gutenberg-Richter frequency-magnitude relation, the *b*-value, the 1-D dynamical lattice model, friction, frictional strength, fractal dimension

I. Introduction

Gutenberg and Richter (1944) reported a frequency-magnitude (FM) relation of earthquakes in the form: $\log N = a - bM$. In this relation, *M* is the earthquake magnitude, and *N* is the discrete or cumulative frequency of the events with magnitudes$\geq M$. The seismic energy, *E*, released during an earthquake is related to *M* in the form: $\log E \sim d \bullet M$. The value of *d* is 1 and 3/2 for small and large earthquakes, respectively (Ekstrom and Dziewonski, 1988). Hence, there is a power-law function between *N* and *E* in the form: $N{\sim}E^{-B}$, where $B=b/d$. The *b*-value varies from region to region and is also dependent upon the period of time used, but it generally ranges from 0.8 to 1.2. The *b*-value is correlated to geotectonics (e.g., Miyamura, 1962; Wang, 1988; Tsapanos, 1990), and its variation before and after a large earthquake has been considered to be an earthquake precursor (e.g., Smith, 1986; Chen *et al*., 1990).

To understand the physics of the scaling of earthquakes, it is essential to study the faulting process of earthquakes, which is very complicated and cannot be completely solved using a simple model. Several factors must be taken into account for modeling. A minimal set of ingredients includes plate tectonics, brittle-ductile fracture rheology, the stress distribution after fracture, the geometry of faults, the friction law, and the healing process from dynamic friction to static friction after a fault stops moving.

Earlier studies on the physical process associated with the *b*-value were based on laboratory work on rock fractures. Mogi (1967) reported the effect of the degree of heterogeneity of the media on the *b*-value. Scholz (1968) correlated the increase in the *b*-value with the decrease in the ambient stress level. Recently, from the fragmentation of materials (Turcotte, 1986a) and the fractal distribution of the strain and stress of the crustal deformation (Turcotte, 1986b), Turcotte studied the causes of the magnitude-frequency relation. King (1983) considered the geometrical origin of the *b*-value based on self-similar fault systems.

Burridge and Knopoff (1967) proposed a dynamical lattice model (abbreviated as the BK model hereafter) to approach fault dynamics. This model has since been applied to dynamically simulate the FM relation of earthquakes (Otsuka, 1972; Yamashita, 1976; Rundle and Jackson, 1977; Cao and Aki, 1984/85, 1986; Carlson and Langer, 1989; Carlson, 1991a, 1991b; Carlson *et al.*, 1991; Knopoff *et al.*, 1992; Schmittbuhl *et al.*, 1996; Shaw, 1995; Shaw *et al.*, 1992; Wang, 1991, 1994 1995, 1996, 1997). Before 1989, the number of mass elements used was usually small; thus, the simulated FM distribution was not good enough. Since 1989, the number of mass elements used has largely increased; thus, the simulated FM distribution is well-defined.

Bak *et al*. (1987,1988) suggested a sandpile (cellular automaton-type) model for the interpretation of the power-law phenomena. This model shows the important property of self-organized criticality (SOC): from any initial state, the system evolves to a critical state characterized by a power-law distribution of activities. On the basis of the concept of SOC, a model mathematically equivalent to the lattice model in the limit of zero-mass of the blocks has also been widely used by numerous authors in studying the FM relation. In addition, the statistical physics models, including the percolation theory, and numerical simulations based on a 3-D quasi-static elastic model have also been used to simulate the FM relation. However, these topics will not be discussed in this work.

In this article, the previously mentioned publications on the scaling of the FM relation of earthquakes based on the 1-D BK dynamical lattice model are reviewed.

II. Theory

The 1-D BK model consists of a chain of N mass elements of equal mass, m, and springs with each mass element being linked by two coil springs of strength, K, with two other neighbors and each mass element also being pulled through a leaf spring of strength, L, by a moving plate with a constant velocity, V. This system is illustrated schematically in Fig. 1. Initially, all mass elements rest in an equilibrium state, and the spacing between two mass elements is 'a'. Each mass element is located at position u_n, measured from its initial equilibrium position, along the x-axis, which is in the direction of motion. Furthermore, each mass element is subjected to a state-dependent frictional force, F_n. The equation of motion at the n-th mass element of the system is in the form:

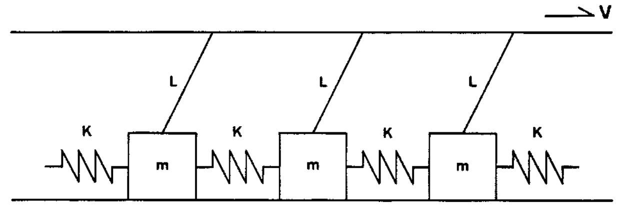

Fig. 1. One-dimensional mass-spring model to simulate earthquake sequences.

$$m(d^2u_n/dt^2)=K(u_{n+1}-2u_n+u_{n-1})-L(u_n-Vt)-F_n. \quad \textbf{(1)}$$

Obviously, the spacing 'a' is not an explicit parameter in Eq. (1). By comparing the BK equation with a finite-difference equation, which is an approximation of a 2-D plain strain type wave equation in the neighborhood of a fault surface, Yamashita (1976) related K to two of Lame's constants (λ and μ) and the ratio of the S-wave velocity, β, and P-wave velocity, α, of the material, i.e., $K=[2(\lambda+\mu)(\beta/\alpha)^2](\delta z/\delta y)$, and L to the rigidity of the material, i.e., $L=\mu(\delta y/\delta z)$. Since the two quantities, i.e., δy and δz, are the spacing units along and perpendicular to the axis of the model, respectively, both $\delta z/\delta y$ and $\delta y/\delta z$ are dimensionless quantities. When the spacings along the two axes are equal, the two parameters are 1; thus, K and L are directly associated with the physical parameters of the material. The real data show that the V value is very small, on the order of 10^{-12} units. In practical computations, in order to reduce computational time in generating a large number of events, a larger value of V is generally used.

The boundary conditions at the ends of the model will affect the computational results. Christensen and Olami (1992a) stated that the scaling exponent depends on the boundary condition. However, their result points to a decrease in the difference in the scaling exponents for free and open boundary conditions as the parameter $s/(4s+1)$ increases. This means that the effect of the boundary condition on the scaling exponent can be ignored when the value of s is large. However, in most studies, a periodic boundary condition has been applied at the two end mass elements in solving Eq. (1).

The equation of motion essentially consists of two processes. The first of these is the coupling between the moving plate and the mass elements through the leaf spring L. The other process is the generation of "self-stress", as it was called by Andrews (1978), which originates from the joint effect of the coil spring K between two mass elements and the leaf spring L. The coil spring K plays a role only in transferring energy from one mass element to another; thus, it does not change the total energy in the system. However, the spring L plays two roles: one is to provide energy to the system from the driving force caused by the moving plate, i.e., the LVt term in Eq. (1), while the other is to take energy from the system. This indicates that the spring, L, can change the total energy of the system. Therefore, the stiffness ratio s ($=K/L$) is a significant parameter representing the level of conservation of energy in the system.

Friction is a very complicated physical process and has been studied for a long time. Dieterich (1972) first stressed the time-dependence of a frictional constant. Ruina (1983) proposed a state-dependent function to describe friction. From laboratory experiments, Dieterich (1979) and Shimamoto (1986) reported that the dynamic friction is velocity-dependent. Essentially, the velocity-dependent friction law includes two processes: the velocity-weakening process when the sliding velocity is smaller than a critical velocity, v_c, and the velocity-hardening process when the sliding velocity is greater than v_c. Burridge and Knopoff (1967) first considered a velocity-dependent, weakening-hardening friction law for dynamic simulation of earthquakes. A displacement hardening-softening friction law for seismicity simulations was used by Cao and Aki (1984/85) in seismicity simulations. Cao and Aki (1986) used a rate- and velocity-dependent friction law for the same purpose and stated that the two friction laws used by them have different effects on simulation results. Rice and Tse (1986) considered a rate- and state-dependent friction law to control the dynamic motion of the mass element of a single degree of freedom system. Some authors (Carlson, 1991a, 1991b; Carlson and Langer, 1989; Carlson *et al*., 1991; Shaw, 1994, 1995; Shaw *et al*., 1992; Schmittbuhl *et al*., 1996) considered a velocity-weakening friction law in the form of $(1+v)^{-1}$, where v is the sliding velocity of a mass element, to control the sliding of the mass element and its variants. The generalized velocity- and state-dependent friction law is rather complicated (Horowitz, 1988). For the first-order approximation, Wang and Knopoff (1991) considered a piece-wise, linear velocity-dependent weakening-hardening friction law for

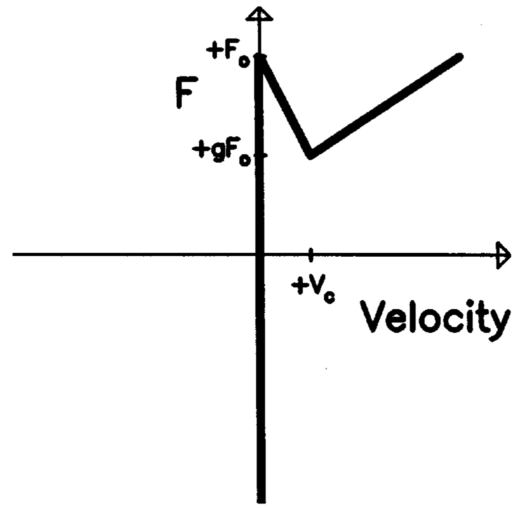

Fig. 2. A linearly velocity-dependent friction law: F_o=the breaking strength; v_c=the critical velocity; and g=the friction drop ratio.

seismicity simulations. Such a friction law (as shown in Fig. 2) takes the form:

$$F(v)=F_o-rv \quad (v<v_c), \tag{2a}$$

$$=gF_o+\gamma v \quad (v>v_c), \tag{2b}$$

where v ($=du/dt$) is the velocity. As shown in Fig. 2, Eq. (2) is defined only for $v>0$; in other words, when the sliding velocity is smaller than zero, the frictional force is a negative infinity. This means that no backward motion is allowed. In Eqs. (2a) and (2b), F_o denotes the breaking strength or static friction strength. The decreasing rate, r, and increasing rate, γ, of the dynamic frictional force with sliding velocity are the two parameters of the model. When $v=v_c$, the dynamic frictional force reaches the minimum value, gF_o, where g is the friction drop ratio and is also a significant parameter of the model. Its value is positive yet smaller than 1. A smaller g value produces a larger force drop, thus resulting in a larger event. Hence, in some sense, the drop in the frictional force from F_o to gF_o behaves like a source supplying additional energy to the mass element for sliding. This friction law was used by Knopoff *et al*. (1992) and Wang (1991, 1994, 1995, 1996, 1997).

In the modeling by Carlson and her co-workers, the distribution of the breaking strengths is almost uniform; thus, the de-localized events, for which all mass elements of the model are in an unstable state, can easily be generated. Nussbaum and Ruina (1987) claimed that the homogeneous fault stress is generally unstable. Nevertheless, it is known that the fault zones where earthquakes occur are usually quite complicated, and that a large earthquake does not occur so often in a fault zone. Seismological and geological observations show that the mechanical properties and geometry of a fault zone are heterogeneous. From laboratory experiments, Mogi (1963) addressed the importance of the inhomogeneity of the material of the fault plane for seismicity and its *b*-value. But, in contrast, based on laboratory results, Scholz (1968) stressed that the state of stress, rather than the heterogeneity of the material, plays the most important role in determining the *b*-value. The breaking strength is the main mechanical parameter reflecting the state of stress over the fault zone for the occurrence of a rupture and is one of the most important properties certain to influence seismicity and its scaling. Das and Aki (1977) and Aki (1979) defined a barrier model and Kanamori and Stewart (1978) defined an asperity model to describe such an inhomogeneous distribution of the breaking strengths over the fault zone for earthquake occurrence. Based on a single rider model, Nur (1978) studied the effect of displacement-dependent or position-dependent friction on a rupture. Although Archambeau (1978) argued against using Nur's oversimplified single rider model to study the complexity of earthquakes, Nur's results nevertheless reveal the influence of the inhomogeneity of the breaking strengths over the fault plane on the propagation of a rupture. He related the rupture velocity to the gradient of the breaking strengths.

An inhomogeneous distribution of the breaking strengths was used by Yamashita (1976), Rundle and Jackson (1977) and Cao and Aki (1984/85, 1986) in seismicity simulations. In practice, different distribution functions can be selected to describe the inhomogeneity of the distribution of the breaking strengths. Field survey results (Scholz and Aviles, 1986; Aviles *et al*., 1987; Okubo and Aki, 1987) and laboratory observations (Brown and Scholz, 1985) have suggested that the geophysical and geometrical properties over the faults have, in general, a fractal distribution. Fractal properties are commonly found in natural phenomena (Mandelbrot, 1982; Turcotte, 1989, 1992). A parameter describing fractal geometry is called a fractal dimension, *D*, as defined by Mandelbrot (1982). Wang (1991) and Wang and Knopoff (1991) first used a fractal function to describe the distribution of the breaking strengths. Since a fractal is a nonlinear phenomenon, the use of a fractal distribution of the breaking strengths makes their model a nonlinear one. They used the Midpoint Displacement Method developed by Saupe (1988) to obtain a fractal distribution. This method can only produce discrete fractals with *N* points, where *N* is $2^{1\text{evel}}+1$, and the parameter 'level' is the computational level needed to produce a finite discrete fractal structure.

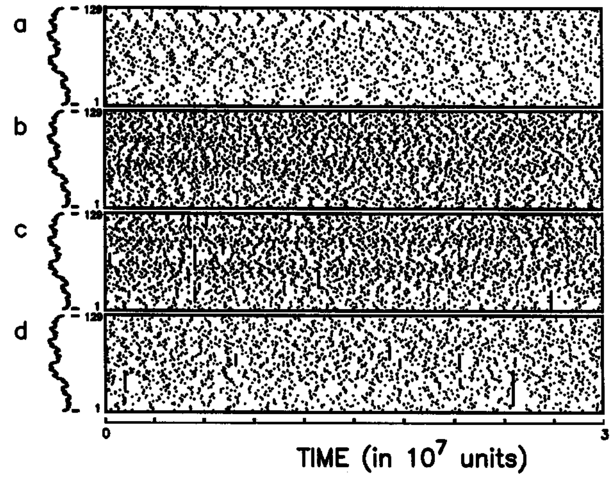

Fig. 3. The space-time patterns of events: (a) for s=10, (b) for s=50, (c) for s=70 and (d) for s=l10. [Reprinted from Wang (1995)]

From a velocity-dependent friction law, Carlson *et al*. (1991) related the magnitude range with a power-law function to a parameter so as to specify the decreasing rate of friction strength with sliding velocity and the stiffness ratio. The action of the leaf spring force between a mass element and the moving plate and friction produces a composite effect, which is actually a dissipation effect, on the earthquake rupture. In accordance with the BK model together with the above-mentioned stepwise linear friction law, i.e., Eq. (2), Wang (1996) defined three types of rupture for velocity-weakening friction: subsonic-type friction when $r>2(Lm)^{1/2}$, sonic-type friction when $r=2(Lm)^{1/2}$, and supersonic-type friction when $r<2(Lm)^{1/2}$.

For a certain mass element, when the sum of the driving force due to the moving plate and spring forces from its neighbors exceeds the breaking strength, it is accelerated and starts to slide. After a while, the increase in either the spring force due to the change in the relative positions of the mass element and its neighbors or in the dynamic frictional force with sliding velocity decelerates the motion. Finally, the mass element stops and sticks, and this results in a drop in the total force. However, the moving plate, which always loads the mass element, increases the total force on the mass element enough to reach the breaking strength, and then to push it to slide again.

The displacement of a mass element is measured from its new equilibrium position to the one where it sticks after sliding. Since several mass elements might slide almost simultaneously within a certain time span; the sum of the displacements of the related mass elements in such a time span provides a time history of the displacements. Such a time history is considered to be an event. An example showing the space-time patterns (abbreviated as the ST patterns) of events for four values of s, i.e., 5, 40, 80, and 120, are displayed in Fig. 3. The line segment linking up the slid mass elements represents an event. The longer line segment consists of a larger number of mass elements and represents a bigger event. Since the seismic energy, E_s, is proportional to the maximum slip $u_{\max}$ and the related force drop, Δf, i.e., $E_s=\Delta f \bullet u_{\max}$, the logarithmic value of the sum of the seismic energy of the mass elements for one event is taken to be the magnitude, M, of the event, i.e., $M=\log(\Sigma E_s)$. Hence, this magnitude is an energy-based magnitude scale rather than the commonly used magnitude scale based on the peak amplitude of the seismogram. Wang used this magnitude in his studies. The magnitude used by Carlson and her co-workers was based on the seismic moment, which is the sum of the displacements of all the mass elements of an event, i.e., $M=\log(\Sigma\Delta u_i)$, and which is different from the above-mentioned one. Thus, for the energy-based magnitude, the scaling exponent of $\log N$ versus M must be similar to 'B' in the relation: $N\sim E^{-B}$ as mentioned previously and different from both 'b' in the Gutenberg-Richter's FM relation and 'b' in the relation used by Carlson and her co-authors. But, for simplicity, the notation 'b' will hereafter be used to express the scaling exponent of the current relation of $\log N$ versus M.

III. The Effects on the Magnitude-Frequency Relation due to Model Parameters

Burridge and Knopoff (1967) first studied the frequency-magnitude problem based on the BK model. They stated that the system exhibits the Gutenberg-Richter-type (GR-type) FM relation, and that the scaling exponent (i.e., the parameter B mentioned previously) of the frequency-energy relation is about 1. Rundle and Jackson (1977) showed that the linear behavior of the FM relation is not an immutable law but rather is dependent on the mechanical properties of the fault.

From analytic and numerical computations, Carlson and Langer (1989) divided the simulation events into three types: microscopic, localized, and de-localized events. Only the distribution of frequency versus magnitude of localized events follows a power-law function. The number of microscopic events is remarkably reduced with decreasing magnitude while the distribution of frequency versus magnitude for such events does not follow a GR-type FM function. In the regime of de-localized events, there is a pronounced peak in the plot of frequency versus magnitude and the GR-type FM relation does not exist. In addition, Carlson and her co-workers addressed the notion that the size of the largest event is associated with the length of the fault. Carlson and Langer (1989) and Carlson *et al.* (1991) stated that the range of magnitudes of localized events is essentially influenced by several factors. These factors include the speed of the moving plate, the stiffness ratio (denoted as l^2 in their papers), and the amount of the initial change in the frictional force from a static to a dynamic one. The first factor indicates the loading rate from the moving plate to the system and is the major source of energy supply. The second factor reflects the dissipation of energy through the coupling between the fault and the moving plate. The third factor also represents the change of energy due to friction. Of course, friction is also responsible for the loss of energy. Therefore, the dissipation of energy in either of the two ways does influence the FM relation. Shaw *et al.* (1992) also obtained similar results.

Based on the 1-D dynamical model with a spatially inhomogeneous distribution of the breaking strengths and a dissipation mechanism due to seismic-wave radiation, Knopoff *et al.* (1992) stated that a self-organizing, spatially localized sequence of seismic events would be constrained by spatial fluctuations. They also stated that the GR-type FM relation is a correlate of the geometry of localization. Schmittbuhl *et al.* (1996) stressed that two distinct regimes in the statistical distribution of event sizes and magnitudes are separated by a characteristic size, L^*, which depends on the elastic stiffness and the dissipation ratio. A characteristic length L_c, related to L^*, controls a cross-over between two different dynamical regimes. For events of size smaller than L_c, the system exhibits scaling laws.

Aki (1982) postulated a positive relation between the b-value and the fractal dimension, D, in the form $D=3b/c$, where c is the slope of the log moment versus the magnitude relation, and c is about 1.5. The theoretical relation between the two parameters is $b=D/3$ according to Turcotte (1986a) and $b=D/2$ according to Turcotte (1986b) based on different models. However, Hirata (1989) and Wang and Lee (1996) reported a negative correlation between the two parameters for earthquakes in Japan and Taiwan, respectively. Hirata (1989) discussed this problem in detail. He concluded that the fractal dimension of the geometry of fault planes used by Aki is a special case of the capacity dimension of either an asperity or a barrier distribution in which all asperities or barriers are connected to each other without isolation, and where the dimension can be regarded as the fractal dimension of the surface of the fault plane. Yet, this is not necessarily true for the

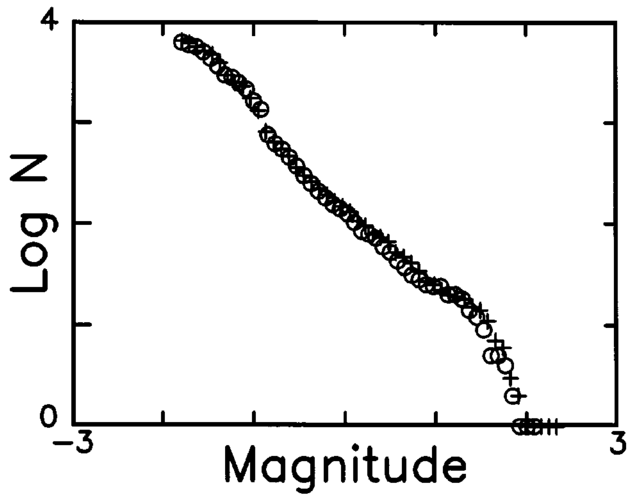

Fig. 4. The plots of log$N$ versus M for two values of fractal dimension, D: '❍' for D=l.1, and '+' for D=l.5. [Reprinted from Wang (1991)]

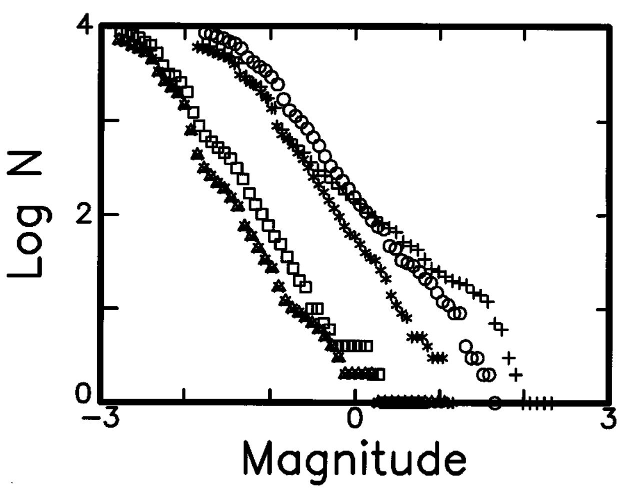

Fig. 5. The plots of log$N$ versus M: (1) '❍' for s=100 and r=1 and 'Δ' for s=50 and r=1 when g=0.8 and F_{omax}=5 units; (2) '❑'for s=100 and r=∞ and '+' for s=50 and r=∞ when g=0.6 and F_{omax}=5 units; and (3) '∗' for s=100 and r=∞ and '×' for s=50 and r=∞ when g=0.8 and F_{omax}=10 units. [Reprinted from Wang (1995)]

observed seismicity produced from various fault planes. Wang (1991) studied the correlation between the b-value and the fractal dimension of the distribution of the breaking strengths from synthetic seismicity based on the 1-D BK model. From simulation results (Fig. 4), he concluded that the b-value is not noticeably dependent upon the D value. Wang's simulation result is different from both the theoretical postulation made by Aki (1982) and Turcotte (1986a, 1986b) and observations (Hirata, 1989; Wang and Lee, 1996). A possible reason for the difference might be that the D used by Wang (1991) concerns the distribution of the breaking strengths of the fault while the D used by the other authors is related to the geometry of the faults.

Carlson and Langer (1989) and Wang (1996) studied the effect on the FM relation due to the variation of the dynamic friction strength. Carlson and Langer (1989) reported that the b-value is affected by a parameter α, which is the ratio of the largest characteristic slipping speed to the speed at which the friction strength is appreciably reduced. The b-value first increases with α and then becomes a constant when α is larger than a certain value. Wang (1995) studied the effects on the FM relation due to the change in the value of the friction drop ratio, g, and the maximum value of the breaking strength, F_{omax} (Fig. 5). Results show that the b-value is larger for g=0.6 (a large friction drop) than for g=0.8 (a small friction drop), and that the b-value does not depend upon the maximum breaking strength. Wang (1996) stated that the weakening rate of the dynamic frictional force with sliding velocity, i.e., r in Eq. (2), remarkably affects the FM relation. For large events, the motion of a mass element is controlled by both weakening and hardening friction, but for small and intermediate-size events, the motion of a mass element is controlled mainly by velocity-weakening friction. Wang (1996) also showed that the b-value decreases with increasing r (Fig. 6). The b-values are different for the above-mentioned three types of friction. The largest b-value is associated with supersonic friction, the intermediate one with sonic friction and the smallest one with subsonic friction. In addition, he also stressed that large r is more capable of localizing the events and prohibiting the generation

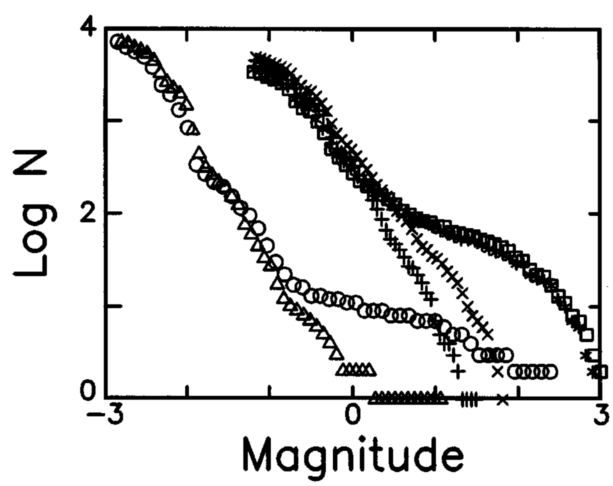

Fig. 6. The plots of log$N$ versus M for five values of r when s=50: 'Δ' for r=1, '×' for r=2, '❑' for r=3, '∗' for r=50, and '❍' for r=∞. Included is the plot (in '+') for r=∞ and s=100. [Reprinted from Wang (1996)]

of de-localized events than small r. Schmittbuhl *et al*. (1996) stated that for friction laws allowing local reversal backslipping, the FM distribution does not exhibit a GR-type FM distribution. On the other hand, when backslipping is precluded, a GR-type distribution is observed. In addition, Shaw (1995) stressed that slip-weakening friction also produces slip complexity, thus leading to FM scaling.

When a mass element stops moving after sliding, the healing of the dynamic friction strength to static friction strength, including the type and delay time of healing, will influence the next rupture. In other words, the consequence of non-instantaneous healing must be significant for seismicity (Rundle and Jackson, 1977). In fact, Cao and Aki (1986) stated that non-instantaneous healing lengthens the time needed for a fault slip to stop, reduces the interaction between different fault segments and, finally, counteracts the smoothing effect. Other than this, the effects of such a non-instantaneous healing process was not included in other papers. Wang (1997) studied the effect of the frictional healing process from dynamic friction to static friction. He defined a ratio h/LV, where h is the frictional healing rate from dynamic friction to static friction and LV is the tectonic loading rate. The plots of log$N$ versus M for three values of h/LV, i.e., ∞, 10^2, 10, and 1, are shown in Fig. 7. His results show that when $h/LV>1$, the FM distributions of model events exhibit a GR-type scaling, and the related b-value is relatively insensitive to h/LV. When $h/LV=1$, the pattern of the FM distribution and the related b-value change somewhat. However, the ratio is only a minor factor in terms of its effect on the FM relation.

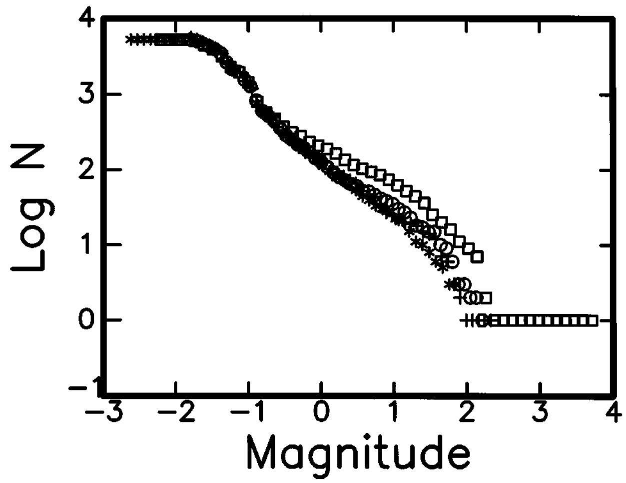

Fig. 7. The plots of log$N$ versus M for three values of $\eta=h/LV$: '+' for $\eta=\infty$, '❍' for $\eta=10^2$, '*' for $\eta=10$, and '❑' for $\eta=1$. [Reprinted from Wang (1997)]

For a slip weakening, single-degree-of-freedom spring-slider model, the stiffness K of the spring is considered to be the most important parameter controlling the instability of the model (see Rice (1979) and Li (1987)). For such a model, small K (less than a critical value) rather than large K can produce an unstable rupture. Based on a simple two-dimensional anti-plane strain softening model, Stuart (1981) considered the ratio K_f/K_s, where K_f and K_s are the stiffnesses of the fault zone and the elastic surroundings, to be a significant indicator of earthquake instability. He stated that instability occurs when the ratio reaches unity. Stuart (1986) redefined K_s/K_f as the stiffness ratio to indicate the instability of the system. Obviously, this parameter is just the ratio between the stiffnesses of the fault zone and the elastic surroundings and cannot directly display the coupling between them unless a constitution law is given to describe the correlation between the fault and the elastic surroundings. Although the stiffness K was considered in a series of works by Stuart and his co-authors (e.g., Stuart and Mavko, 1979; Stuart *et al*., 1985; Stuart, 1986, 1988), they did not study in depth the effect of the variation in K on the earthquake rupture. In numerous studies using the cellular automaton iteration, the l/s value was designated to be zero, i.e., $L=0$, (cf. Ito, 1992) or a very small value (Brown *et al*., 1991). In those studies, the effect due to the coupling between a mass element and the moving plate was mostly ignored. In this kind of modeling, the system can self-organize itself very easily, and it shows SOC. However, in some other studies, the effect of the variation of the stiffness ratio on the FM relation was included. Based on a simulation for a two degree-of-freedom earthquake model, Nussbaum and Ruina (1987) addressed the importance of the stiffness ratio (called the coupling ratio in their paper) on the slip pattern.

From analytic computations, Carlson and her co-authors stated that the difference between the upper and lower bound magnitudes of the magnitude range, within which the power law holds, is proportional to $\log(s^{3/2})$. Simulation results obtained by Carlson (199la) showed that the b-values for three values of s, i.e., 36, 64, and 144, are mostly around -1.0 and independent of s. For the 2-D BK model, using the cellular automaton iteration, Huang *et al*. (1992) also obtained an almost constant b-value of about 1.36 for five values of s, i.e., 10, 15, 20, 30, and 40. Nakanishi (1990) calculated the distribution of frequency versus magnitude for four values of s, i.e., 2.00, 2.83, 4.50 and 9.50, using the cellular automaton iteration. Although he did not calculate the actual value of b, his results still showed an increase in the b value with s. Based on the results of cellular-automaton modeling for the isotropic 2-D models, Christensen and Olami (1992a, 1992b) stated that the scaling exponent decreases continuously as a function of $s/(4s+1)$. Although this parameter does not change very much when the s value is large, it does decrease with increasing s. Hence, three different relations between the b-value and the stiffness ratio exist.

According to the friction law described by Eq. (2), with a fractal distribution of breaking strengths, Wang (1994,1995) integrated Eq. (1) numerically for the models for various values of s from 5 to 130 when D=1.5, level=7, F_o=5 units, $r=\infty$, γ=1, m=l unit and L=l unit. When level=7, the model consists of 129 mass elements. It is evident from Fig. 3 that different values of s produce different ST patterns. For small s (for instance 10), the number of slid mass elements of an event is generally small; in contrast, for large s (for instance 110), many larger events with longer line segments appear. This indicates that for large s, a larger number of mass elements can be driven to an unstable state during a time span, thus leading to a bigger event. In the four cases, the number of events in the range with strong breaking strengths is evidently smaller than that in the range with weak breaking strengths.

The plots of frequency versus magnitude for six values of s, i.e., 10, 30, 50, 70, 90, and 110, are shown in Fig. 8 (Wang, 1995). The data points for s=10, and also for s<10, cannot be described completely by only using a single regression line in a predominant magnitude range. It seems appropriate to interpret the data points using two regression lines or a curve. It can be seen from Fig. 8 that the data point related to M=0 (denoted as M_c) is almost the intersecting point of all FM distributions with different values of s. When $M<M_c$, the log$N$ value, to some extent, decreases with

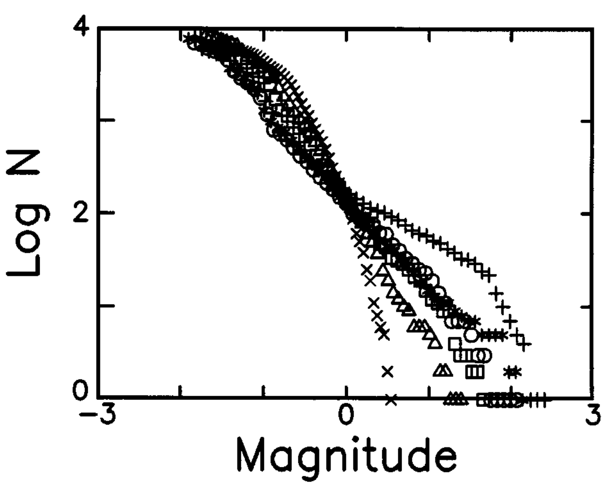

Fig. 8. The plots of log$N$ versus M for six values of s: '+' for s=l10, '❍' for s=90, '*' for s=70, '❑' for s=50, 'Δ' for s=30, and '×' for s=10. [Reprinted from Wang (1995)]

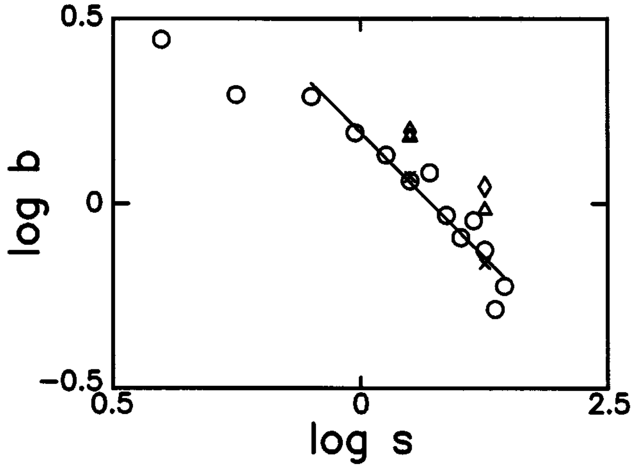

Fig. 9. Plot of b versus s in the logarithmic scale. The circles represent the b value in the cumulative frequency-magnitude relation for the models with a friction law with $r=\infty$. The slope value of the solid line ($20 \le s \le 120$) is about −2/3. For s=50 and 100, the triangles show the data point for the model with r=1; the diamond denotes the data point for the model with g=0.6; the cross represents the data point for the model with F_{omax}=10 units. [Reprinted from Wang (1995)]

increasing s. Of course, the difference between any two log$N$ values for a certain value of M is small. In contrast, when $M>M_c$, the value of log$N$ increases with s. Nevertheless, the plots of log$N$ versus M for the cases with values of s in the range of 30 to 100 are somewhat close to each other. Results indicate that the change of s causes opposite effects on the ruptures for large and small events. Small s represents a weaker coupling between mass elements and can only make a smaller number of mass elements slide, thus leading to a larger number of smaller events and a smaller number of bigger events. On the other hand, large s indicates a stronger coupling between mass elements and can cause a larger number of mass elements to slide almost simultaneously, thus resulting in a larger number of bigger events and a smaller number of smaller events.

Unlike the results presented by Carlson and Langer (1989), in the regime of large events in Wang (1995, 1996), not only is there an absence of any de-localized event, but the number of large events also decreases with M. Meanwhile, the regime of microscopic events in Wang (1995, 1996) is not so significant as that given by Carlson and Langer (1989). Nonetheless, in the regime of events with medium magnitude, which is between the microscopic and de-localized regimes, the data points of log$N$ versus M are distributed very closely around a line. Figure 8 shows that the lower bound of the magnitude range is almost constant whereas the upper bound of the magnitude range increases with s. This increase in the upper bound magnitude leads to

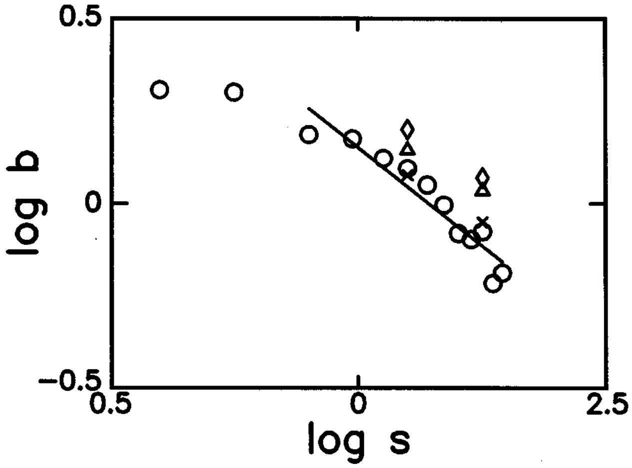

Fig. 10. Plot of b versus s in the logarithmic scale. The circles represent the b value in the discrete frequency-magnitude relation for the models with a friction law with $r=\infty$. The slope value of the solid line ($20\le s\le 120$) is about $-1/2$. For $s=50$ and 100, the triangles show the data point for the model with $r=1$; the diamond denotes the data point for the model with $g=0.6$; the cross represents the data point for the model with $F_{omax}=10$ units. [Reprinted from Wang (1995)]

a larger magnitude range with a power-law function.

Wang (1994, 1995) also explored the possible correlation between the b-value and stiffness ratio s. The plots of b versus s are shown in Fig. 9 in the cumulative frequency-magnitude relation and in Fig. 10 in the discrete frequency-magnitude relation. Wang (1995) stressed that from simulation events, the b-value of the cumulative frequency-magnitude relation is less than that of the discrete frequency-magnitude relation. This is similar to the result mentioned by Main (1992) based on observed results. For both plots, the data points for $s>20$ are distributed around a line very well, and that those with $s<20$ depart from the linear trend. The regression line for the data points for $s>20$ has a slope value of about $-2/3$ as shown in Fig. 9 and $-1/2$ as shown in Fig. 10. The related regression lines are individually plotted in Figs. 9 and 10, with a solid line. Obviously, there is a power-law correlation between b and s: $b\sim s^{-2/3}$ for the cumulative frequency-magnitude relation and $b\sim s^{-1/2}$ for the discrete frequency-magnitude relation. Included also in Figs. 9 and 10 are the triangles showing the data points for the case with $r=1$ and the diamonds denoting the data points for the case with $g=0.6$ when $s=50$ and 100. Although these data points are above the data points in the models with $r=\infty$, they are still distributed in a linear trend somewhat parallel to the solid line. This seems to suggest that for the friction laws with different values of r and g, the b-values at a certain value of s are different, but the relations of b versus s are almost similar. This correlation of b and s is quite different from those given by Nakanishi (1990), Carlson and Langer (1989) and Carlson *et al*. (1991) for 1-D models. But it is consistent with that given by Christensen and Olami (1992a, 1992b) and Olami *et al*. (1992) for the 2-D models through cellular automaton iteration. Of course, the decreasing function of b versus s in Christensen and Olami (1992a, 1992b) and Olami *et al*. (1992) is not the same as that in Wang (1994, 1995).

There is a lack of observed information capable of directly demonstrating the effect of the stiffness ratio on the FM relation. Nevertheless, Wang (1995) stressed that some significant information can still be implied. The great and apparently systematic variation in the size of large interplate earthquakes in subduction zones (Kanamori, 1971) of the extensional type, such as the Marianas arc, to the highly compressional type, such as the Chilean arc (Uyeda and Kanamori, 1979), reflect different degrees of stiffness of the fault zone and the coupling between a subduction plate and a fault zone between the continental and oceanic plates. Ruff and Kanamori (1980) defined a parameter as a seismic coupling coefficient to distinguish arcs. The seismic coupling coefficient ranges from essentially nil in the Mariana arc to nearly one in the Chilean arc. They assumed that variations in seismic coupling among arcs are due to variations in the mean normal stress that acts across the subduction-zone plate interface. They also stated that the mean normal stress is proportional to the force across the interface. Jarrard (1986) related seismic coupling to more parameters, but he found the strongest to be that found by Ruff and Kanamori (1980). Astiz *et al*. (1988) also addressed the importance of the coupling between two plates on intraplate earthquakes. In the BK model, the force across the interface is represented by a leaf spring. Hence, the spring constants K and L are two major parameters which display the seismic coupling of the model; therefore, the stiffness ratio ($s=K/L$) must be a significant parameter influencing the FM relation. However, it is noted that this definition of the stiffness ratio is different from the one given by Stuart (1986).

IV. Summary

Studies on the b-value of Gutenberg-Richter-type frequency-magnitude or frequency-energy relation and the effects on the b-value due to model parameters based on the one-dimensional dynamic Burridge-Knopoff model (Burridge and Knopoff, 1967) have been reviewed. The frequency-magnitude relation as well as its scaling exponent, i.e., the b-value, is obviously affected by the weakening rate of the dynamic frictional force along with the sliding velocity, the friction drop ratio and the stiffness ratio of the model. However, the fractal dimension of the distribution of the breaking strengths is not a significant parameter affecting the *b*-value. The *b*-value of the cumulative frequency-magnitude relation is less than that of the discrete frequency-magnitude relation. When $20 \leq s \leq 120$, the relation between the *b*-value and the stiffness ratio, *s*, is quite robust. However, the scaling exponents of the relations between the two parameters for the cumulative frequency and the discrete frequency are different.

Acknowledgment

The author would like to express his thanks to Prof. Leon Knopoff of the Institute of Geophysics and Planetary Physics, UCLA, U.S.A., for switching his attention to this very significant research topic. Without long-term financial support by the National Science Council and Academia Sinica, I could not have completed numerous studies on this topic.

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一維動力格點模型用於地震次數-規模關係式之研究

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摘　要

基於芭芮吉和納伯夫的一維動力格點模型（Burridge - Knopoff Model），配以速度相依之摩擦力，許多學者研究了古騰堡-芮赫特型的次數-規模關係式。本文回顧這些有關的研究成果。影響這個關係式和它的尺度指數（即所謂的b值）的主要參數為彈性強度比(s)、動摩擦力隨速度變化的減少率(r)、摩擦力下降比值(g)、破壞強度之非均勻分佈的程度和最大破壞強度(F_{omax})。對某些作者而言，破壞強度的分佈呈碎形，因此碎形度便為一參數。

主要的結果顯示，模擬出來的地震可分三類，即微觀地震、區域化地震和超區域化地震。區域化地震顯示古騰堡-芮赫特型的次數-規模關係式，但其他兩類則否。區域化地震的規模範圍明顯受彈性程度比值影響。關係式本身及b值很明顯受到摩擦律、減少率(r)、摩擦力下降比值(g)和最大破壞強度(F_{omax})的影響，但受到碎形度(D)的影響較小。累加次數-規模關係式的b值比個別次數-規模關係式的b值小。b值和s值間呈冪數相關：對累加次數-規模關係式而言，為b~$s^{-2/3}$；而對個別次數-規模關係式，則為b~$s^{-1/2}$。這兩個相關的指數值與r、g和F_{omax}無關。