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Constructing Analytical Energy Functions for Lossless Network-Reduction Power System Models: Framework and New Developments

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ABSTRACT

The task of an constructing energy function is essential for direct stability analysis of electric power systems. This paper presents a general procedure for constructing analytical energy functions for detailed lossless network-reduction power system stability models. The main contributions of this paper are: (1) it develops canonical representations for lossless network-reduction power system models and shows that such canonical representations cover existing stability models; (2) it derives theoretical results regarding the existence of analytical energy functions for the canonical representations; (3) it presents a systematic procedure for constructing corresponding energy functions.

Key Words: transient stability, energy functions, direct methods, lossless network-reduction power system models

I. Introduction

There exists a rich history of direct stability analysis based on energy function approaches. Development of energy functions of electric power systems can be traced back to Magnusson in the forties (Magnusson, 1947), and this goal was pursued in the fifties by Aylett (1958), in the sixties by Gless (1966) and El-Abiad and Nagappan (1966), and in the seventies by Williems (1971). The survey papers by Ribbens-Pavella and Evans (1985), by Varaiya *et al.* (1985), by Fischl *et al.* (1988), and by Chiang *et al.* (1995), and books by Pai (1989), by Fouad and Vittal (1991), and by Ribbens-Pavella and Murthy (1994) give expositions of recent results and methods concerning direct transient stability analysis.

The accuracy of stability assessments highly depends on the model precision of the underlying power systems. Two different classes of power system models for direct transient stability analysis have been proposed: network-reduction models and network-preserving models (Chiang *et al.*, 1995). Traditionally, direct methods have been developed for network-reduction models, where all of the load is expressed in constant impedances, and the entire network representation is reduced to the generator internal buses. Network-preserving models have been proposed in the last decade to overcome some shortcomings of the network-reduction models and to improve the modeling of generators, exciters, automatic voltage regulators and load representations. Please refer to Chiang, Chu, and Cauley's work for recent developments (Chiang *et al.*, 1995; Chu, 1996).

Construction of analytical energy functions is the central problem in direct stability analysis. Unfortunately, there does not exist a general formulation of analytical energy functions for power system stability models. The objective of this paper is to present a general framework for constructing analytical energy functions for lossless network-reduction power system stability models. To this end, we develop canonical representations for stability models and show that such canonical representations include existing stability models as special cases. We then conduct an analytical study on the developed canonical representations. The canonical representations facilitate the construction of analytical energy functions. Given a new stability model, the proposed canonical representations provide a systematic way to construct an energy function for the new model.

II. Various Network-Reduction Power System Models

This section reviews several existing network-reduction power system models and presents a general representation which includes these existing models. Throughout this paper, we consider a power system consisting of n generators. Buses #1, ..., #n are generator buses, and Bus #n+1 is the reference bus.

1. The Classical Model

In this model, a synchronous machine is represented by a constant voltage behind its transient reactance (i.e., the flux linkages are assumed to be constant during the transient period). Mathematically, the dynamical behaviors of the i-th generator can be represented, using the infinite bus as a reference, by the so-called second-order *swing equations*:

$$\dot{\delta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + P_{m_i} - P_{e_i} \quad i=1,\ 2,\ \ldots,\ n,$$

where

$$P_{e_i} = \sum_{j \neq i}^{n+1} V_i V_j B_{ij} \sin(\delta_i - \delta_j) + \sum_{j \neq i}^{n+1} V_i V_j C_{ij} \cos(\delta_i - \delta_j).$$

V_i is the constant voltage behind the direct axis transient reactance. M_i is the generator's moment of inertia. D_i is the generator's damping. B_{ij} and C_{ij} represent the admittance and the transfer conductance of the i–j element in the reduced admittance matrix of the system, respectively. P_{m_i} is the mechanic power input to the i-th generator.

2. The One-Axis Generator Model

To consider the effects of field flux decay, the one-axis generator model uses one equivalent circuit for the field winding of the rotor. As a result, the voltage behind the direct transient reactance is no longer assumed to be constant. Sasaki first included such a model for direct stability analysis (Sasaki, 1979). The dynamics of each generator are then described by the following equations:

$$\dot{\delta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + P_{m_i} - \sum_{j=1, i \neq j}^{n} V_i V_j (B_{ij} \sin\delta_{ij} + C_{ij} \cos\delta_{ij})$$

$$\frac{T'_{doi}}{x_{di} - x'_{di}} \dot{V}_i = \frac{1}{x_{di} - x'_{di}} E_{fdi} - \frac{1 - (x_{di} - x'_{di})B_{ii}}{x_{di} - x'_{di}} V_i$$

$$+ \sum_{j=1, i \neq j}^{n} V_j (B_{ij} \cos\delta_{ij} + C_{ij} \sin\delta_{ij}), \quad (1)$$

where $\delta_{ij} = \delta_i - \delta_j$, T'_{doi} is the direct axis transient open-circuit time constant, x_{di} is the direct synchronous reactance, and x'_{di} is the direct synchronous transient reactance.

3. The One-Axis Generator with a Simplified Exciter Model

When the exciter control action is included in the generator model, additional differential equations are needed to account for it. The following first-order simplified exciter model has been used:

$$T_{vi} \dot{E}_{fdi} = -E_{fdi} - \mu_i V_{ti} + l_i,$$

where T_{vi} is the time constant of AVR, μ_i is the feedback gain of AVR and l_i is a constant gain used to adjust the location of the desired operating points. If the terminal voltage V_{ti} of each generator has a linear relationship with its quadratic component V_{tqi} (i.e., $V_{ti} = k_i V_{tqi}$, where k_i is a positive constant), then by using the relationship $V_{tqi} = x'_{di} I_{di} + V_i$ (see Fig. 1), the generator dynamics can be expressed as (Miyagi and Bergen, 1986; Pai, 1989)

$$\dot{\delta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + P_{m_i} - \sum_{j=1, i \neq j}^{n} V_i V_j (B_{ij} \sin\delta_{ij} + C_{ij} \cos\delta_{ij})$$

$$\gamma_i \dot{V}_i = -\alpha_i V_i + \beta_i E_{fdi} + \sum_{j=1, i \neq j}^{n} V_j (B_{ij} \cos\delta_{ij} + C_{ij} \sin\delta_{ij})$$

$$\xi_i \dot{E}_{fdi} = -\eta_i V_i - \phi_i E_{fdi} - \sum_{j=1, i \neq j}^{n} V_j (B_{ij} \cos\delta_{ij} + C_{ij} \sin\delta_{ij})$$

$$+ l_i, \quad \textbf{(2)}$$

where $\alpha_i = -B_{ii} + \frac{1}{x_{di} - x'_{di}}$, $\beta_i = \frac{1}{x_{di} - x'_{di}}$, $\gamma_i = \frac{T'_{doi}}{x_{di} - x'_{di}}$, $\eta_i = B_{ii} + \frac{1}{x'_{di}}$, $\phi_i = \frac{1}{\mu_i k_i x'_{di}}$, and $\xi_i = \frac{T_{vi}}{\mu_i k_i x'_{di}}$ are constants. Here,

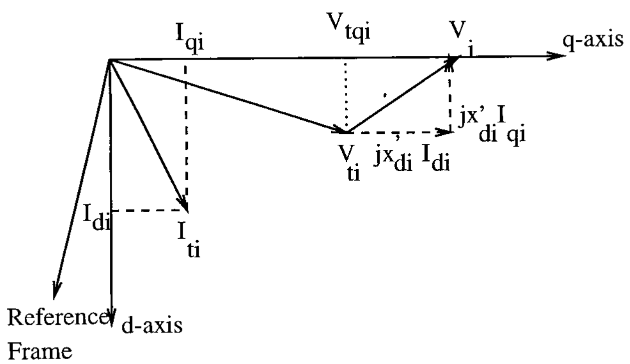

Fig. 1. The phasor diagram of the one-axis generator with the simplified exciter model.

we follow the IEEE recommended notation (Pai, 1989), which is somewhat different from that of Miyagi and Bergen (1986) where industry notation is used.

III. Compact Representations of Lossless Network-Reduction Models

A compact representation of the network-reduction power system model will be presented. It will be shown that the above three network-reduction power system models with *lossless* transmission networks can be rewritten in the following compact form:

$$T\dot{x} = -\frac{\partial}{\partial x} U(x, y)$$

$$\dot{y} = z$$

$$M\dot{z} = -Dz - \frac{\partial}{\partial y} U(x, y), \tag{3}$$

where $x \in I\!R^n$, y and $z \in I\!R^m$, and T, M and D are positive diagonal matrices. The smooth function $U(x, y)$ satisfies the following conditions:

(1) $\nabla U(x, y) = \nabla U(x, y_1 + 2k_1\pi, \ldots, y_n + 2k_n\pi)$ for all $k_i \in Z$, $i=1, \ldots, n$.

(2) For i, $k=1, \ldots, n$ and $j=1, \ldots, m$, there exist polynomials, $p_{1j}(x_1, \ldots, x_m)$, $p_{2ik}(x_1, \ldots, x_m)$, and $p_{3ij}(x_1, \ldots, x_m)$ with positive coefficients such that

$$\left|\frac{\partial}{\partial y_i} U(x, y)\right| \le p_{1i}(|x_1|, \ldots, |x_m|)$$

$$\left|\frac{\partial}{\partial^2 y_i y_k} U(x, y)\right| \le p_{2ik}(|x_1|, \ldots, |x_m|)$$

$$\left|\frac{\partial}{\partial x_j \partial y_i} U(x, y)\right| \le p_{3ij}(|x_1|, \ldots, |x_m|),$$

respectively.

Three lossless network-reduction power system models mentioned in Section II can be put into the compact representation **(3)** and satisfy the conditions (1) and (2). The corresponding variables x, y, z and the function $U(x, y)$ are summarized in Table 1. This result can be verified by direct algebraic manipulations. In Section V, we will demonstrate that the most comprehensive model, the one-axis generator with the simplified exciter model **(3)**, can be put into such a compact representation and satisfies conditions (1) and (2).

The task of deriving an energy function for network-reduction power system models with nonzero transfer conductance of the reduced Y-bus matrix is challenging. Considerable efforts have been concentrated on determining the global energy function analytically. Unfortunately, these efforts, based on either the classical Lure-Postnikov-type Lyapunov function approach or the first integral approach, have been in vain. Narasimhamurthi (1984) has shown that attempts to accommodate line losses by smooth transformation of the energy functions for power systems without losses do not lead to a local Lyapunov function. Moreover, other asymptotic behaviors, such as periodic orbits, have also been discovered and examined in the lossy network-reduction power system model by Abed and Varaiya (1984) and Alexander (1986). All of these results indicate some negative aspects of constructing the global analytical energy function for a lossy network-reduction power system model.

IV. Analytical Energy Functions of Lossless Network-Reduction Power Systems

In this section, we will first discuss some fundamental properties of system **(3)**. We will then present a general procedure of constructing analytical energy functions for lossless network-reduction power system models.

Theorem IV.1: (Hyperbolicity of Equilibrium Points) Consider the following system:

$$\dot{x} = -\frac{\partial}{\partial x} U(x, y)$$

$$\dot{y} = -\frac{\partial}{\partial y} U(x, y). \tag{4}$$

If all of the equilibrium points of system **(4)** are hyperbolic, then all the equilibrium points of system **(3)** are hyperbolic. Moreover, $(x_e, y_e, 0)$ is a type-k equilibrium point of system **(3)** if and only if (x_e, y_e) is a type-k equilibrium point of the system **(4)**.

Proof: See Appendix.

Having shown the static properties of the equilibrium points of the compact representation of network-reduction power system models **(3)**, next we will study the existence of the energy function for such a compact representation **(3)**. Recall the definition of the energy functions: we say that a function $V: I\!R^n \rightarrow I\!R$ is an energy function for a given nonlinear system $\dot{x} = f(x)$ if the following three conditions are satisfied:

Table 1. Various Network-Reduction Power System Models and Corresponding Potential Energy Functions

Model	System Equations	x	y	z	Potential Energy Function $V(x, y)$
Classical Generator Model	$\dot{\delta}_i = \omega_i$ $M_i\dot{\omega}_i = -D_i\omega_i + P_{m_i} - P_{ei}$ $P_{ei} = \Sigma_{j=1}^{n+m+1} V_iV_jB_{ij}\sin(\delta_i - \delta_j)$	ϕ	δ	ω	$-\Sigma_{i=1}^{n}\Sigma_{j=i+1}^{n} V_iV_jB_{ij}\cos\delta_{ij} + P_{m_i}\delta_i$
One-Axis Generator Model	$\dot{\delta}_i = \omega_i$ $M_i\dot{\omega}_i = -D_i\omega_i + P_{m_i} - \Sigma_{j=1,i\neq j}^{n} V_iV_j B_{ij}\sin\delta_{ij}$ $\frac{T'_{doi}}{x_{di}-x'_{di}}\dot{V}_i = \frac{1}{x_{di}-x'_{di}}E_{fdi} - \frac{1-(x_{di}-x'_{di})B_{ii}}{x_{di}-x'_{di}}V_i + \Sigma_{j=1,i\neq j}^{n} V_j B_{ij}\cos\delta_{ij}$	V	δ	ω	$-\Sigma_{i=1}^{n}\Sigma_{j=i+1}^{n} V_iV_jB_{ij}\cos\delta_{ij} + P_{m_i}\delta_i + \frac{1}{x_{di}-x'_{di}}V_iE_{fdi} + \frac{1-(x_{di}-x'_{di})B_{ii}}{2(x_{di}-x'_{di})}V_i^2E_{fdi} - \frac{1-(x_{di}-x'_{di})B_{ii}}{x_{di}-x'_{di}}V_i + \Sigma_{j=1,i\neq j}^{n} V_jB_{ij}\cos\delta_{ij}$
One-Axis Generator Plus First Order AVR Model	$\delta_i = \omega_i$ $M_i\dot{\omega}_i = -D_i\omega_i + P_{m_i} - \Sigma_{j=1,i\neq j}^{n} V_iV_j B_{ij}\sin\delta_{ij}$ $\gamma_i\dot{V}_i = -\alpha_iV_i + \beta_iE_{fdi} + \Sigma_{j=1,i\neq j}^{n} B_{ij}V_j\cos\delta_{ij}$ $\xi_iE_{fdi} = -\eta_iV_i - \phi_iE_{fdi} - \Sigma_{j=1,i\neq j}^{n} B_{ij}V_j\cos\delta_{ij} + l_i$	V E_{fd}	δ	ω	$-\Sigma_{i=1}^{n}\Sigma_{j=i+1}^{n} V_iV_jB_{ij}\cos\delta_{ij} + P_{m_i}\delta_i + \frac{1}{x_{di}-x'_{di}}V_iE_{fdi} + \frac{1-(x_{di}-x'_{di})B_{ii}}{2(x_{di}-x'_{di})}V_i^2E_{fdi} - \frac{1-(x_{di}-x'_{di})B_{ii}}{x_{di}-x'_{di}}V_i + \Sigma_{j=1,i\neq j}^{n} V_j B_{ij}\cos\delta_{ij} + \frac{1}{2}\mathbf{V}^TC^{-1}A\mathbf{V} - \mathbf{V}^TC^{-1}L$ where $A = \mathrm{diag}\begin{bmatrix} \frac{\alpha_i}{\gamma_i} & -\frac{\beta_i}{\gamma_i} \\ \frac{\eta_i}{\xi_i} & \frac{\phi_i}{\xi_i} \end{bmatrix}_{i=1}^{n}$ $C = \mathrm{diag}\begin{bmatrix} \frac{1}{\gamma_i} & -\frac{1}{\xi_i} \\ -\frac{1}{\xi_i} & s_i \end{bmatrix}_{i=1}^{n}$ $L = \left[0, \frac{l_i}{\xi_i}\right]_{i=1}^{n}$ $\mathbf{V} = [V_i, E_{fdi}]_{i=1}^{n}$

(1) The derivative of the energy function $V(x)$ along any system trajectory $x(t)$ is non-positive, i.e.

$$\dot{V}(x(t)) = \frac{\partial}{\partial x} V(x)f(x) \le 0 .$$

(2) If $x(t)$ is a non-trivial trajectory (i.e. $x(t)$ is not an equilibrium point (e.p.)), then there does not exist a time interval, say $[t_1, t_2]$, $t_2 > t_1$, such that $\dot{V}(x(t)) = 0$ for $t \in [t_1, t_2]$. Mathematically, this can be expressed as follows: along any non-trivial trajectory $x(t)$, the set

$$\{ t \in R : \dot{V}(x(t)) = 0 \}$$

has measure zero in $\mathbb{R}$.

(3) If a trajectory $x(t)$ has a bounded value of $V(x(t))$ for $t \in \mathbb{R}^+$, then the trajectory $x(t)$ is also bounded. Stating this in brief:

If $V(x(t))$ is bounded, then $x(t)$ is also bounded.

Property (1) indicates that the energy is non-increasing along its trajectory but does not imply that the energy is strictly decreasing along its trajectory. There may exist a time interval $[t_1, t_2]$ such that $\dot{V}(x(t))=0$ for $t \in [t_1, t_2]$. Properties (1) and (2) imply that the energy is strictly decreasing along any system trajectory. Property (3) states that, along any system trajectory, the energy function is a proper map, but that its energy need not be a proper map for the entire state space. Recall that a proper map is a function $f: X \to Y$ such that, for each compact set $D \in Y$, the set $f^{-1}(D)$ is compact in X (Abraham *et al.*, 1988). Obviously, an energy function is not a Lyapunov function.

The following theorem provides a sufficient condition for the existence of an energy function for the compact representation of the network-reduction power system models.

Theorem IV.2: (Existence of an Energy Function)

For the compact representation of the network-reduction power system model **(3)**, consider the function W: $\mathbb{R}^{n+2m} \rightarrow \mathbb{R}$:

$$W(x,y,z) = K(z) + U(x,y) = \frac{1}{2} z^T Mz + U(x,y). \quad \textbf{(5)}$$

Suppose that along every nontrivial trajectory $(x(t), y(t), z(t))$ with bounded function value $W(\bullet, \bullet, \bullet)$, $x(t)$ is also bounded. Then, $W(x, y, z)$ is an energy function for system **(3)**.

Proof: See Appendix.

Remarks:

In Theorem IV.2, a sufficient condition for the existence of an energy function for the compact representation **(3)** is provided. Also, an explicit analytical energy function is presented. It is worth mentioning that:

(1) The analytical energy function itself can be written as the sum of two separate functions: the *kinetic energy* function $K(z)=\frac{1}{2} z^T Mz$ and the *potential energy* function $U(x, y)$. Such formulations agree with a recent development of direct methods for transient stability analysis in which the controlling u.e.p. is detected via an artificial, dimensional-reduction system whose energy function is composed of $U(x, y)$ only (Chiang *et al.*, 1987, 1994, 1995; Chiang and Chu, 1995; Chu, 1996).

(2) The sufficient condition in Theorem IV.2 is equivalent to condition (3) of the energy function. However, it provides a more convenient form for verification. Moreover, the sufficient condition can be further simplified by exploring the special structure of the underlying power system models. To elaborate on this point, we next work on the one-axis generator with a simplified exciter model and present a procedure for constructing an analytical energy function for the model.

V. Analytical Energy Functions for the One-Axis Generator with a Simplified Exciter Model

The one-axis generator with a simplified exciter network-reduction model is an aggregation of the one-axis generator model and first-order exciter dynamics. Our idea of constructing an energy function for this class of network-reduction power system models is to combine the energy function of the one-axis generator model **(1)** with other terms introduced by the exciter dynamics. First, consider the variables internal voltage V_i and exciter voltage E_{fdi} together. Define $\mathbf{V}_i=(V_i, E_{fdi})$ and

$$U(V,\delta) = -\sum_{i=1}^{n}\sum_{j=i+1}^{n} V_i V_j B_{ij} \cos\delta_{ij}.$$

Since for $i=1, \ldots, n$, the function $U(V, \delta)$ has the following properties:

$$\frac{\partial U(V,\delta)}{\partial V_i} = -\sum_{j=1, i\neq j}^{n} B_{ij} V_j \cos\delta_{ij}$$

$$\frac{\partial U(V,\delta)}{\partial E_{fdi}} = 0.$$

The voltage dynamics $\mathbf{V}_i$ at each generator i can also be expressed as

$$\dot{\mathbf{V}}_i = -A_i \mathbf{V}_i - C_i \frac{\partial U(V,\delta)}{\partial \mathbf{V}_i} + L_i,$$

where

$$A_i = \begin{bmatrix} \frac{\alpha_i}{\gamma_i} & -\frac{\beta_i}{\gamma_i} \\ \frac{\eta_i}{\xi_i} & \frac{\phi_i}{\xi_i} \end{bmatrix}$$

$$C_i = \begin{bmatrix} \frac{1}{\gamma_i} & -\frac{1}{\xi_i} \\ -\frac{1}{\xi_i} & s_i \end{bmatrix}$$

$$L_i = [\,0, \frac{l_i}{\xi_i}\,]^T. \quad \textbf{(6)}$$

Thus, the element s_i in C_i can be arbitrarily chosen because $\frac{\partial U(V,\delta)}{\partial E_{fdi}}=0$. Here, s_i is a free parameter and can be chosen to be larger than $\frac{\gamma_i}{\xi_i^2}$ such that C_i is a symmetric positive definite matrix.

We will next consider the overall dynamical equation. Let

$\mathbf{V}=(\mathbf{V}_1, \ldots, \mathbf{V}_n)^T$

A=block diag $[A_1, \ldots, A_n]$

C=block diag $[C_1, \ldots, C_n]$

$L=[L_1, \ldots, L_n]^T$.

Define

$$\mathrm{U}(\mathbf{V},\delta) = \frac{1}{2}\mathbf{V}^T C^{-1} A\mathbf{V} - \mathbf{V}^T C^{-1} L - P_m^T \delta + U(V,\delta).$$

It is easy to see that $\mathbf{U}(\mathbf{V}, \delta)$ is a modification of the potential energy function $-P_m^T\delta + U(V, \delta)$ of the one-axis generator model with the new items introduced by the simplified exciter model. Since the function $\mathbf{U}(\mathbf{V}, \delta)$ has the properties

$$\frac{\partial \mathbf{U}(\mathbf{V}, \delta)}{\partial \delta_i} = \frac{\partial U(V, \delta)}{\partial \delta_i} - P_{m_i}$$

$$\frac{\partial \mathbf{U}(\mathbf{V}, \delta)}{\partial \mathbf{V}} = C^{-1}A\mathbf{V} - C^{-1}L + \frac{\partial U(V, \delta)}{\partial \mathbf{V}}, \tag{7}$$

the overall dynamical equation **(2)** can also be expressed as

$$C^{-1}\dot{\mathbf{V}} = -\frac{\partial \mathbf{U}(\mathbf{V}, \delta)}{\partial \mathbf{V}}$$

$$\dot{\delta} = \omega$$

$$M\dot{\omega} = -D\omega - \frac{\partial \mathbf{U}(\mathbf{V}, \delta)}{\partial \delta}. \tag{8}$$

From the formulation of Eq. (8), it is clear that Eq. (8) also belongs to the compact representation of the network-reduction power system **(3)**. It remains to check that the potential energy function $\mathbf{U}(\mathbf{V}, \delta)$ satisfies conditions (1) and (2). First, since the overall dynamical equation **(2)** is periodic with respect to the variable δ, it is easy to see that condition (1) is automatically satisfied. Applying the partial derivative to Eq. (7), we get the following equations:

$$\left|\frac{\partial \mathbf{U}(\mathbf{V}, \delta)}{\partial \delta_i}\right| = \left|-\sum_{j=1, i\neq j}^{n} B_{ij}V_j\cos\delta_{ij} - P_{mi}\right|$$

$$\leq \sum_{j=1, i\neq j}^{n} B_{ij}|V_j| + |P_{mi}|$$

$$= p_{1i}(|V_1|, \ldots, |V_n|)$$

$$\left|\frac{\partial \mathbf{U}(\mathbf{V}, \delta)}{\partial^2 \delta_i \delta_k}\right| = |-B_{ik}V_iV_k\sin\delta_{ik}|$$

$$\leq B_{ik}|V_i||V_k|$$

$$= p_{2ik}(|V_1|, \ldots, |V_n|)$$

$$\left|\frac{\partial \mathbf{U}(\mathbf{V}, \delta)}{\partial V_j \partial \delta_i}\right| = |-B_{ij}V_j\cos\delta_{ij}|$$

$$\leq B_{ij}|V_j|$$

$$= p_{3ij}(|V_1|, \ldots, |V_n|).$$

Clearly, $p_{1i}(|V_1|, \ldots, |V_n|)$, $p_{2ik}(|V_1|, \ldots, |V_n|)$, and $p_{3ij}(|V_1|, \ldots, |V_n|)$ are polynomials with positive coefficients. Thus, condition (2) is also satisfied.

We will next present a sufficient conditions for the existence of an analytical energy function and construct an explicit energy function by exploring the special structure of the one-axis generator with the simplified exciter model. Note that the dynamics of the voltage variable $\mathbf{V}$ can also be expressed as the following equations:

$$\dot{\mathbf{V}} = -(A + C\Sigma(\delta))\mathbf{V} + L, \tag{9}$$

where

$$\Sigma(\delta) = \begin{bmatrix} B_{11} & 0 & B_{12}\cos\delta_{12} & 0 & \cdots & B_{1n}\cos\delta_{1n} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ B_{12}\cos\delta_{12} & 0 & B_{22}\cos\delta_{22}' & 0 & \cdots & B_{2n}\cos\delta_{2n} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ B_{1n}\cos\delta_{12} & 0 & B_{2n}\cos\delta_{2n} & 0 & \cdots & B_{nn}\cos\delta_{nn} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

By examining the special structure of the vector field of Eq. (9), the following theorem can be derived.

Theorem V.1 (Existence of Analytical Energy Functions for One-axis Generator with Simplified Exciter Models)

Consider the one-axis generator with the simplified exciter model **(2)**. Define

$$W(\mathbf{V}(t), \delta(t), \omega(t))$$

$$= \frac{1}{2}\omega^T M\omega + \mathbf{U}(\mathbf{V}, \delta)$$

$$= \frac{1}{2}\omega^T M\omega + \frac{1}{2}\mathbf{V}^T C^{-1}A\mathbf{V} - \mathbf{V}^T C^{-1}L - P_m^T\delta$$

$$-\sum_{i=1}^{n}\sum_{j=i+1}^{n} V_iV_jB_{ij}\cos\delta_{ij},$$

where A, C, and L are defined by Eq. (6). Suppose that along any nontrivial trajectory ($\mathbf{V}(t)$, $\delta(t)$, $\omega(t)$) of system **(2)** with a bounded value of $W(\mathbf{V}(t), \delta(t), \omega(t))$, the set $\Gamma=\{t\in \mathbb{R}^+: \det(A+C\Sigma(\delta(t)))=0\}$ has a measure zero; then $W(\mathbf{V}(t), \delta(t), \omega(t))$ is an analytical energy function for system **(2)**.

Proof: See Appendix.

Remarks:

(1) Mathematically, the sufficient condition in Theorem V.1 ensures that the boundness of $\mathbf{V}(t)$ can be guaranteed by the boundness of $\dot{\mathbf{V}}(t)$. Since $\mathbf{V}(t)$ is bounded, based on Theorem III.2, it follows that $W(\mathbf{V}(t), \delta(t), \omega(t))$ is indeed an analytical energy function for system **(2)**.

(2) For general nonlinear systems, there exist some state variables which are bounded but whose derivatives are unbounded. Please refer to Desoer and Vidyasagar (1975) for more concrete examples. However, the sufficient condition presented in Theorem V.1 is not too restrictive since the free parameters s_t in C matrix can be employed to simplify the condition $\det(A+C\Sigma(\delta))\neq 0$.

IV. Conclusion

We have developed a general framework for constructing analytical energy functions of network-reduction power system models. In particular, we have

(1) developed canonical representations for network-reduction power system models and shown that such canonical representations cover existing stability models,

(2) derived theoretical results regarding the existence of analytical energy functions for the canonical representations,

(3) constructed an analytical energy function for network-reduction models.

Appendix

Proof of Theorem IV.1:

First, all of the equilibrium points of system **(3)** can be characterized by the following set:

$$E=\{(x,y,z): \frac{\partial}{\partial x}U(x,y)=0,\ z=0,\ -Dz-\frac{\partial}{\partial y}U(x,y)=0\}$$

$$=\{(x,y,0): \nabla U(x,y)=0\}.$$

To prove this result, we will use the Sylvester's Inertia Theorem (Wimmer, 1974). The Jacobian of system **(3)** evaluating at the equilibrium point (x_e, y_e, 0) is

$$J(x_e,y_e,0)$$

$$=\begin{bmatrix} T^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & M^{-1}\end{bmatrix}\begin{bmatrix} -\frac{\partial}{\partial^2 x}U(x_e,y_e) & -\frac{\partial}{\partial x\partial y}U(x_e,y_e) & 0 \\ 0 & 0 & I \\ -\frac{\partial}{\partial x\partial y}U(x_e,y_e) & -\frac{\partial}{\partial^2 y}U(x_e,y_e) & -D\end{bmatrix}$$

$$=AB.$$

Since all of the critical values of $U(x, y)$ are regular, i.e., $\nabla^2U(x_e, y_e)$ is nonsingular, B and $J(x_e, y_e, 0)$ are also nonsingular. Let

$$H(x_e, y_e)=\text{block diag}\ [-\nabla^2U(x_e,y_e), -M^{-1}].$$

It is clear that $H(x_e, y_e)$ is a nonsingular Hermitian matrix, and that

$$J(x_e,y_e,0)H(x_e,y_e)+H(x_e,y_e)J^T(x_e,y_e,0)$$

$$=\begin{bmatrix} 2T^{-1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2M^{-1}DM^{-1}\end{bmatrix}\geq 0.$$

Applying the Sylvester inertia theorem, we conclude that $\text{In}(H(x_e, y_e)) = \text{In}(J(x_e, y_e, 0))$. Therefore, all the equilibrium points of system **(3)** are hyperbolic, and

$$n_u(J(x_e,y_e,0))=n_u(H(x_e,y_e))=n_u(\hat{J}(x_e,y_e)),$$

where $\hat{J}(\bullet, \bullet)$ denotes the Jacobian matrix of system **(4)**. This result also indicates the second statement of the theorem. This completes the proof.

Proof of Theorem IV.2:

We will check conditions (1)-(3) of the energy function:

(1) Differentiating $W(x(t), y(t), z(t))$ along the trajectory:

$$\dot{W}(x(t),y(t),z(t))=\frac{\partial W^T}{\partial x}\dot{x}+\frac{\partial W^T}{\partial y}\dot{y}+\frac{\partial W^T}{\partial z}\dot{z}$$

$$=-\frac{\partial U^T}{\partial x}T^{-1}\frac{\partial U}{\partial x}-z^TDz\leq 0. \quad \textbf{(10)}$$

This inequality shows that condition (1) of the energy function is satisfied.

(2) Suppose that there is an interval $t\in[t_1, t_2]$ such that $\dot{W}(x(t),y(t),z(t))=0$; hence, $z(t)=0$ and $\dot{x}(t)=0$ for $t\in[t_1, t_2]$. However this also implies that $y(t)$ is a constant. It then follows that the system is at an equilibrium point; hence, condition (2) of the energy function also holds.

(3) Since $x(t)$ is bounded along the nontrivial trajectory with bounded $W(\bullet, \bullet, \bullet)$, only components $y(t)$ and $z(t)$ will be examined in condition (3). We will use Barbalat's Lemma and the covering map to attack this problem.

(i) From Eq. (10), it follows that

$$\dot{W}(x(t),y(t),z(t))\leq -a|\dot{x}(t)|^2$$

$$\dot{W}(x(t),y(t),z(t))\leq -b|z(t)|^2$$

for some positive numbers a and b, respectively. Since along the system trajectory, $W(x(t), y(t), z(t))$ is monotonically decreasing and bounded:

$$W(x(\infty),\ y(\infty),\ z(\infty))=\lim_{t\to\infty} W(x(t),\dot{y}(t),z(t))$$

exists, and

$$W(x(\infty),\ y(\infty),\ z(\infty))-W(x(0),\ y(0),\ z(0))$$

$$=\int_0^\infty \dot{W}(x(t),y(t),z(t))dt \le -a\int_0^\infty |\dot{x}(t)|^2 dt$$

$$W(x(\infty),\ y(\infty),\ z(\infty))-W(x(0),\ y(0),\ z(0))$$

$$=\int_0^\infty \dot{W}(x(t),y(t),z(t))dt \le -b\int_0^\infty |z(t)|^2 dt.$$

Therefore, $\dot{x}(t)$ and $z(t)\in L_2[0,\ \infty)$. Since $\dot{x}(t)$ and $z(t)$ are both continuous, they are also bounded.

(ii) From the hypothesis that $x(t)$ is bounded, the system dynamic equation **(3)** indicates that $\dot{z}(t)$ is also bounded. Since $z(t)\in L_2[0,\ \infty)\cap L_\infty[0,\ \infty)$ and $\dot{z}(t)\in L_\infty[0,\ \infty)$, by Barbalat's lemma (Sastry and Bodson, 1989), $z(t)\to 0$ as $t\to\infty$.
Differentiating $\dot{z}(t)$, from system **(3)**,

$$M\ddot{z} = -D\dot{z} - \frac{\partial}{\partial^2 y}U(x,y)z - \frac{\partial}{\partial x\partial y}U(x,y)\dot{x}.$$

By assumption **(3)**, both $\frac{\partial}{\partial^2 y}U(x,y)$ and $\frac{\partial}{\partial x\partial y}U(x,y)$ are bounded. Therefore, $\ddot{z}(t)\in L_\infty[0,\ \infty)$. Since $z(t)\to 0$ as $t\to\infty$, by Rudin (1976), $\dot{z}(t)\to 0$ as $t\to\infty$. From system equation **(3)**, it follows that $\frac{\partial}{\partial y}U(x,y)\to 0$ asymptotically.

(iii) Because of the periodic structure of the variable y, we can define a covering map

$$\pi_y : \mathbb{R}^{m+n}\to \mathbb{R}^m \underbrace{\times[0,2\pi]\times\ldots\times[0,2\pi]}_{n}$$

and a continuous function

$$\overline{U}: \mathbb{R}^m \underbrace{\times[0,2\pi]\times\ldots\times[0,2\pi]}_{n} \to \mathbb{R}$$

such that $U(x,\ y)$ is a lifting of $\overline{U}(\pi_y(x,y))$, i.e., $\overline{U}(\pi_y(x,y))=U(x,\ y)$ for all $(x,\ y)\in \mathbb{R}^{m+n}$. Since $\frac{\partial}{\partial y}U(x,y)\to 0$, with respect to the covering map π_y, $\frac{\partial}{\partial y}\overline{U}(\pi_y(x,y))\to 0$. Also, $\overline{W}(x,y,z)=W(\pi_y(x,y,z))$ satisfies conditions (1) and (2) of the energy function for the restriction dynamics on $\mathbb{R}^m \underbrace{\times[0,2\pi]\times\ldots\times[0,2\pi]}_{n}$. Note that, under the restriction space $\mathbb{R}^m \underbrace{\times[0,2\pi]\times\ldots\times[0,2\pi]}_{n}$, $\pi_y(y(t))$ is bounded since $\underbrace{[0,2\pi]\times\ldots\times[0,2\pi]}_{n}$ is compact. By condition (1) of the energy function $\overline{W}(x,y,z)$, every boundary trajectory $(x(t),\ \pi_y(y(t)),\ z(t))$ will converge to one of the equilibrium points. That is, $\pi_y(y(t))$ will converge to a point y^* asymptotically. Therefore, there exists a $T>0$, a neighborhood N of y^* such that $\pi_y(y(t))\in N$ for $t>T$. Now, consider the preimage $\pi_y^{-1}(N)$ of N; it is the union of the disjoint open set $\{U_\alpha\}$ in $\mathbb{R}^n$. Since $\dot{y}(t)\in L_\infty[0,\ \infty)$ and the projected trajectory $\pi_y(y(t))$ is path connected and, hence, connected, $y(t)$ can not pass more than one disjoint open set of $\{U_\alpha\}$ in an arbitrarily small time. In other words, after $t>T$, $y(t)$ will belong to only one of the disjoint open sets, say U_{α^*}. Therefore, $y(t)$ is also bounded.

From parts (1)-(3), $x(t)$, $y(t)$ and $z(t)$ are bounded, and $W(x(t),\ y(t),\ z(t))$ is an energy function.

Proof of Theorem V.1:

From Theorem IV.2, it follows that the proof is essentially simplified to show that, along every nontrivial trajectory with a bounded energy function, the voltage magnitude **V** is also bounded. We will explore the special structure of the vector field of the one-axis generator with the simplified exciter model. From Eq. (10), it follows that $\dot{\mathbf{V}}$ is also bounded along every nontrivial trajectory with a bounded energy function. Since

$$\mathbf{V} = (-A - C\Sigma(\delta))^{-1}(\dot{\mathbf{V}} - L),$$

if $\det(-A-C\Sigma(\delta))$ has a measure on $\mathbb{R}^+$, then except for a finite point on $\mathbb{R}^+$, **V** exists and is also bounded. This completes the proof.

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無損耗網路簡化模式下電力系統能量函數之建立：架構及其新發展

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摘　要

以直接法分析電力系統暫態穩定度中，最困難的問題在於能量函數的建立。本文將嘗試在無損耗網路簡化模式之電力系統下，提出可解析能量函數的建立程序。本文主要的結果可歸納如下：(1)發展無損耗網路簡化模式電力系統的正規方程式，並且說明此正規方程式可涵蓋現有之電力系統網路無損耗簡化模式。(2)針對此正規方程式，推導可解析能量函數之存在性。(3)提出一系統程序以建立其相關的可解析能量函數。