Outflow Distribution along Multiple-port Diffusers

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ABSTRACT

The outflow distribution along a multiple-port diffuser was explored in this study by employing analytical, numerical and experimental methods. The effects of two dimensionless parameters controlling the flow distribution, i.e., the wall friction parameter (α) and the port momentum parameter (β), were evaluated. When the wall friction parameter is negligible, the port discharge increases downstream, and the flow distribution becomes more uneven as the value of the port momentum parameter increases. On the other hand, the port discharge decreases downstream when the momentum parameter is negligible. Analytical solutions for the port outflow distribution were derived for the cases in which either the wall friction parameter or the port momentum parameter is dominant. The numerical solutions agree well with both analytical solutions and experimental data. As for uniformity of the outflow distribution of all ports, it is also found that these two parameters should be controlled within a suitable range (i.e., $\beta \equiv 0.5940\alpha$), and that the diameter of the diffuser should be close to $\frac{0.353}{2 - \gamma_d}fL$.

Key Words: diffuser, port discharge, flow distribution

I. Introduction

Multiple-port diffusers, often referred to as manifolds, are used in sprinkling infiltration systems (McNown, 1954), gas pipe burners (Keller, 1949), thermal discharges (Vigander *et al.*, 1970) and ocean outfalls (Rawn *et al.*, 1961; Lee and Yau, 1996). The port discharge along a multiple-port diffuser depends on the pressure difference across the port as well as on the geometry of the port. The two main goals in diffuser hydraulics are, first, to obtain a uniform flow distribution for all the ports, and secondly to minimize the total head loss in the pipe system (Fischer *et al.*, 1979).

There are two types of approaches to studying diffuser hydraulics. One is the energy approach, based on the energy equation for a flowing fluid inside the diffuser and calculation of the flow rate backward by assuming a given head at the downstream end (Rawn et al., 1961; Fischer et al., 1979). The other is the momentum approach, which solves the flow momentum equation for the flowing fluid and finds the flow distribution inside the manifold directly (e.g., Bajura (1971) and Shen (1992)). The energy approach solves for the flow distribution in a stepwise manner, thus retaining flexibility in handling spatially geometric changes, such as in the sizes of the ports and the diffuser, the inverse slope, and the density of the discharging fluid. However, it is difficult to explore the combined effects of individual physical parameters by adopting the energy approach since it is based on a trial-anderror algorithm. On the other hand, the momentum approach can evaluate the effects of different parameters by assuming

fixed geometric conditions and a continuous lateral flow.

Previous researches on the uniformity of port discharges mostly focused on the effects of specified parameters, such as the size and shape of the port, and the diameter and wall friction coefficient of the diffuser pipe. For example, the ratio of the total port area to the cross area of the diffuser pipe should be in the range of 1/3 to 1/2 (Fischer et al., 1979). Either increasing the port discharge coefficient or reducing the wall friction of the pipe will make the distribution of the port discharge more even (Vigander et al., 1970). However, the effects of the aforementioned parameters on the port discharge are not independent of one another, and their combined effect should be quantified systematically. In this paper, the dimensionless parameters controlling the distribution of the port discharge are analyzed by employing both theoretical analysis and numerical simulations. An experimental study was also performed to verify the analytical and numerical results. Such dimensionless parameters are useful in hydraulic design of multiple-port diffusers.

II. Theoretical and Numerical Analysis

The lateral discharge from a porous diffuser with a sufficient number of ports can be assumed to be continuous since the port interval is generally less than one-tenth the total length of the diffuser. For a diffuser with a constant diameter and with a closed downstream end, as shown in Fig. 1, the steady axial (x-direction) momentum equation for an isothermal, incompressible fluid in the diffuser has the following form

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(Bajura, 1971):

$$\frac{1}{\rho}\frac{dP}{dx} + \frac{f}{2D}U^2 + 2U\frac{dU}{dx} + \gamma_d UV\frac{\lambda\pi D}{A} = 0, \qquad (1)$$

where ρ = the density of the fluid, P = the fluid pressure in the diffuser, f = the friction coefficient of the diffuser pipe, D = the diameter of the diffuser, U = the cross-sectional average flow velocity in the diffuser, γ_d = the hydrostatic pressure recovery coefficient for the port discharge, V = the velocity of the port discharge, A = the cross-sectional area of the diffuser pipe ($A = \frac{\pi}{4}D^2$), and λ = the porosity, defined as the ratio of the total port area ($\lambda = \frac{n\pi d^2}{4}$) to the surface area of the diffuser (πDL), i.e.,

$$\lambda = \frac{nd^2}{4DL},$$
 (2)

where n = the port number, d = the port diameter, and L = the diffuser length.

The first term in Eq. (1) is the hydrostatic pressure gradient, the second term the wall friction, the third term the axial momentum change of the diffuser before and after the port, and the last term the momentum flux carried out by the port discharge. Based on the equation of continuity, the lateral outflow velocity per unit length can be written as

$$V = -\frac{A}{\lambda \pi D} \frac{dU}{dx} \,. \tag{3}$$

The energy equation at the diffuser-port junction shown in Fig. 1 is written as

$$(P - P_r) = \rho (1 + C_{td} + f_1 \frac{\ell}{d}) \frac{V^2}{2} = \rho H \frac{V^2}{2}, \qquad (4)$$

where P_r = the fluid pressure in the lateral pipe, C_{td} = the headloss coefficient at the junction, f_1 = the friction coefficient of the lateral pipe, ℓ = the length of the lateral pipe with port opening (see Fig. 1), and $H = 1 + C_{td} + f_1 \frac{\ell}{d}$.

By scaling the flow velocity and the distance with the upstream cross-sectional velocity in the diffuser, $U_{in} (= Q_{in}/A)$, and the diffuser length, *L*, respectively, i.e., $\tilde{U} = U(x)/U_{in}$, $\tilde{x} = x/L$, one can derive a dimensionless flow distribution equation in the diffuser from Eqs. (1), (3) and (4) as follows (Shen, 1992):

$$\frac{d}{d\tilde{x}}\left[\left(\frac{d\tilde{U}}{d\tilde{x}}\right)^2 + \left(\beta\tilde{U}\right)^2\right] + \left(\alpha\tilde{U}\right)^2 = 0$$
(5a)

with

$$\alpha = \sqrt{\frac{fL}{DH}} Ar , \qquad (5b)$$

$$\beta = \sqrt{\frac{2 - \gamma_d}{H}} Ar , \qquad (5c)$$



Fig. 1. A schematic sketch of a multiple-port diffuser.

where the wall friction parameter, α , depicts the frictional effect of the diffuser pipe, the port momentum parameter, β , represents the port momentum effect, and *Ar* denotes the ratio of the port area as

$$Ar = \frac{nd^2}{D^2}.$$
 (5d)

According to Eqs. (5b) and (5c), the wall friction parameter, α , reflects the combined effects of the wall roughness, diffuser length, diffuser diameter, local head loss at the port, and the aspect ratio of the port to the diffuser. The port momentum parameter, β , is determined based on the port geometry, local head loss at the port, and the aspect ratio of the port to the diffuser. In the case of a port without a lateral pipe (i.e., $\ell = 0$ in Fig. 1), the contraction effect at the port will be encountered, i.e.,

$$Ar = \frac{nd^2}{D^2} C_d , \qquad (5e)$$

where C_d = the contraction coefficient of the port, which is defined as

$$C_d = \frac{q}{a\sqrt{2gE}},$$
(5f)

q = the port discharge, a = the port area ($a = \frac{1}{4}\pi d^2$), E = the specific energy inside the diffuser pipe ($E = \frac{P}{\gamma} + \frac{U^2}{2g}$), where P = the pressure, and γ = the specific weight of the fluid ($\gamma = \rho g$).

The boundary conditions for Eq. (5a) are given as $\tilde{U}(\tilde{x}=0)=1$ and $\tilde{U}(\tilde{x}=1)=0$. From now on, all the superscripts for the dimensionless velocities \tilde{U} and \tilde{V} will be dropped for simplicity. Since Eq. (5a) is a nonlinear ordinary differential equation, it is quite difficult to obtain an analytical solution. However, Shen (1992) presented a solution for Eq. (5a), without details, as follows:

$$U = \exp(-c_1 \tilde{x}) \frac{\sin(c_2(1-\tilde{x}))}{\sin c_2},$$
 (6)

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$$V = \exp(-c_1 \tilde{x}) [\cos(c_2 \tilde{x}) + \tan(c_2) \frac{1 - \frac{c_1}{c_2} \cot(c_2)}{1 + \frac{c_1}{c_2} \tan(c_2)} \sin(c_2 \tilde{x})]$$
(7a)

where

$$A = \left[-\frac{\alpha^2}{4} + \left(\frac{\alpha^4}{16} + \frac{\beta^6}{27}\right)^{1/2}\right]^{1/3},$$
 (7b)

$$B = -\left[\frac{\alpha^2}{4} + \left(\frac{\alpha^4}{16} + \frac{\beta^6}{27}\right)^{1/2}\right]^{1/3},$$
 (7c)

$$C_1 = \frac{(A+B)}{2}, \qquad (7d)$$

$$C_2 = \frac{\sqrt{3}}{2}(A - B)$$
. (7e)

A numerical test showed that Eq. (6) does not agree with Eq. (5a) (Cheng, 1997). Consequently Eq. (6) is not a valid solution. Since Eq. (5a) is governed by the two parameters α and β , the effect of these two parameters will be discussed first based on theoretical analysis of special cases, and will later be systematically analyzed based on the results of numerical simulations.

1. The Perturbation Method for Port Flow Distribution

If the frictional effect is relatively small, such as for a short, smooth diffuser, the value of α is close to zero. When $\alpha = 0$, Eq. (5a) is reduced to a simpler form:

$$\frac{d}{dx}(U_0'^2 + \beta^2 U_0^2) = 0.$$
 (8)

Equation (8) allows us to explore the effect of the port momentum parameter, β , on the port discharges. Based on the boundary conditions, $U_0(0) = 1$ and $U_0(1) = 0$, one can easily find the solution (Bajura, 1971):

$$U_o(\tilde{x}) = \frac{\sin\beta(1-\tilde{x})}{\sin\beta}.$$
 (9)

According to Eq. (3), the dimensionless port discharge, $V_i(\tilde{x})$, that is, the discharge at any location \tilde{x} with respect to the upstream port discharge, is defined as

$$V_i(\tilde{x}) = \frac{V_0(\tilde{x})}{V_0(0)} = \frac{U'_0(\tilde{x})}{U'_0(0)} = \frac{\cos\beta(1-\tilde{x})}{\cos\beta}.$$
 (10)

According to Eq. (10), the port discharge increases downstream under the condition that the friction loss is completely neglected (i.e., $\alpha = 0$). The maximum port discharge thus occurs at the downstream end, with a value of $V_i(1) =$ $\frac{1}{\cos\beta}$. The flow distribution becomes more uneven with the increase of the port momentum parameter, β , since the value of $V_i(1)$ increases accordingly.

If the value of α is small, its effect on the flow distribution can be evaluated by using the perturbation method. The firstorder solution, with respect to α , for the flow velocity in the diffuser pipe is written as

$$U(\tilde{x}) = U_0(\tilde{x}) + \alpha^2 U_1(\tilde{x}),$$
(11)

where $U_0(x)$ is the zero-order solution, as shown in Eq. (9), and U_1 is the correction term due to the existence of α .

Substituting Eq. (11) into Eq. (5a) and neglecting order terms higher than α^2 , one can obtain

$$U_{1}''(\tilde{x}) + \beta^{2} U_{1}(\tilde{x}) = -\frac{U_{0}^{2}(\tilde{x})}{2U_{0}'(\tilde{x})} = \frac{\{\sin[\beta(1-\tilde{x})]\}^{2}}{2\beta \sin\beta \cos[\beta(1-\tilde{x})]}$$
(12)

with the boundary conditions

$$U_1(0) = 0, U_1(1) = 0.$$
 (13)

Let

$$U_1(\tilde{x}) = U_h(\tilde{x}) + U_p(\tilde{x}), \qquad (14a)$$

where

$$U_h(\tilde{x}) = c_1 \cos(\beta \tilde{x}) + c_2 \sin(\beta \tilde{x}), \qquad (14b)$$

$$U_{p}(\tilde{x}) = k_{1} \cos(\beta \tilde{x}) + k_{2} \sin(\beta \tilde{x}), \qquad (14c)$$

and

$$k_{1}(\tilde{x}) = \frac{-1}{2\beta^{2} \mathrm{sin}\beta} \int_{0}^{x} \frac{\mathrm{sin}^{2}[\beta(1-\tilde{x})]\mathrm{sin}(\beta\tilde{x})}{\mathrm{cos}[\beta(1-\tilde{x})]} d\tilde{x}$$
$$= \frac{-1}{2\beta^{2}} A(\tilde{x}) - \frac{\mathrm{cos}\beta}{2\beta^{3}\mathrm{sin}\beta} B(\tilde{x}), \qquad (14d)$$

$$A(x) = \left[\frac{1}{2}(1-\tilde{x}) - \frac{1}{4}\sin(2\beta) + \frac{1}{4\beta}\sin(2\beta\tilde{x})\right],$$
 (14e)

$$B(\tilde{x}) = \left[\ln\cos\beta - \frac{1}{2}\cos^2\beta - \ln\cos(\beta\tilde{x}) + \frac{1}{2}\cos^2(\beta\tilde{x})\right].$$
(14f)

Similarly,

$$k_{2}(\tilde{x}) = \frac{1}{\beta} \int_{0}^{\tilde{x}} \frac{\sin^{2}[\beta(1-\tilde{x})]\cos(\beta\tilde{x})}{2\beta\sin\beta\cos[\beta(1-\tilde{x})]} d\tilde{x}$$
$$= \frac{\cos\beta}{2\beta^{2}\sin\beta} A(\tilde{x}) - \frac{1}{2\beta^{3}} B(\tilde{x}).$$
(14g)

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By implementing boundary conditions, i.e., Eq. (13), one can find the coefficients k_1 , k_2 , c_1 and c_2 . The derivatives of the correction term U_1 at the boundaries are

$$U'_{1}(1) = \frac{1}{8\beta^{2}} \left[\frac{4\sin^{2}\beta}{\cos\beta} - \frac{[2\beta - \sin(2\beta)]}{\sin\beta} - \frac{8(2\ln\cos\beta + \sin^{2}\beta)\cos\beta}{\sin^{2}\beta} \right],$$
 (15)

$$U'_{1}(0) = \frac{-(2\ln\cos\beta + \sin^{2}\beta)(\sin^{2}\beta + 4\cos^{2}\beta)}{4\beta^{2}\sin^{2}\beta}.$$
 (16)

In order to quantify the port discharge distribution, a uniformity parameter, V_r , is defined as the ratio of the port discharges between downstream and upstream ends, i.e.,

$$V_r = V_i(1) = \frac{U'(1)}{U'(0)} = \frac{U'_0(1) + \alpha^2 U'_1(1)}{U'_0(0) + \alpha^2 U'_1(0)}.$$
 (17)

From Eqs. (9), (15) and (16), one can obtain

$$\int_{1}^{0} \frac{dU}{\sqrt[3]{-\frac{\alpha^2}{2}U^3 + 3c}} = 1.$$
 (22)

The constant *c* in Eq. (22) is solely determined by α . Since an explicit presentation between *c* and α is not available, theoretically, one can obtain the *c* versus α relationship by means of numerical methods, such as Romberg integration. The regression relationship between *c* and α , with a correlation coefficient exceeding 0.99, reads as

$$c = 0.0195\alpha^2 + 0.0237\alpha - 0.3387.$$
 (23)

The uniformity coefficient, V_r , for the diffuser thus reads as

$$V_r = \frac{U'(1)}{U'(0)} = \sqrt[3]{\frac{1}{1 - \frac{\alpha^2}{6c}}}.$$
 (24)

The uniformity coefficient, V_r , based on Eqs. (23) and

$$V_{r} = \frac{U'(1)}{U'(0)} = \frac{1 + \alpha^{2} \left\{ \frac{1}{8\beta^{3}} \left[[2\beta - \sin(2\beta)] + \frac{8(2\ln\cos\beta + \sin^{2}\beta)\cos\beta}{\sin\beta} - \frac{4\sin^{3}\beta}{\cos\beta} \right] \right\}}{\cos\beta + \alpha^{2} [\frac{1}{4\beta^{3}} \frac{(2\ln\cos\beta + \sin^{2}\beta)(\sin^{2}\beta + 4\cos^{2}\beta)}{\sin\beta}]}{(18)}$$

On the other hand, if the port momentum parameter, β , is small relative to the wall friction parameter, α , such as for a long diffuser, one can approach Eq. (5) by assuming that $\beta = 0$, i.e.,

$$2U'U'' + \alpha^2 U^2 = 0. \tag{19}$$

The solution for Eq. (19) is

$$U' = \sqrt[3]{-\frac{\alpha^2}{2}U^3 + 3c} .$$
 (20)

According to the boundary conditions at $\tilde{x} = 0$, U(0) = 1, and $\tilde{x} = 1$, U(1) = 0, one can integrate Eq. (20) as follows:

$$\int_{1}^{U(\tilde{x})} \frac{dU}{\sqrt[3]{-\frac{\alpha^2}{2}U^3 + 3c}} = \tilde{x}$$
(21)

(24), thus, depends on the value of α . As the value of α increases, such as with an increase of the friction coefficient or the diffuser length, the port discharge at the downstream end will be less than that at the upstream end.

2. Numerical Analysis of the Port Flow Distribution

The shooting method is employed to solve Eq. (5a) by transforming the boundary value problem into the initial value problem (Cheng, 1997). By trying an initial slope, i.e., the derivative of the velocity in the diffuser, at the upstream end, one can utilize the fourth order Runge-Kutta method to find the flow velocity as well as its derivative stepwise along the flow. The initial slope will then be modified according to the shooting method if the predicted flow rate at the downstream end is greater than the specified tolerance (10^{-7}) (Press *et al.*, 1986).

III. Experimental Measurement

An acrylic diffuser pipe of 2 m in length and 2 cm in

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with



Fig. 2. A schematic sketch of multiple-port outflow measurement.

diameter was installed horizontally in the Environmental Fluid Laboratory of National Central University. There are 20 lateral ports, each one made 5 mm in diameter at 10-cm intervals by means of careful drilling and surface polishing work, along the longitudinal direction. No lateral pipes are connected to the lateral ports, i.e., $\ell = 0$. The inlet flow has a maximum Reynolds number of 90000. Upstream of the inlet, the flow passes through a straight PVC tube 2 m long with the same diameter in order to reduce the flow disturbance. The experimental setup is shown in Fig. 2. The pressure in the diffuser pipe was measured by using a pressure transducer (Druck Ltd., Leicester, U.K., model PDCR910, pressure range = 100 Kpa, error range = 1 Pa). Prior to measuring the port discharge, the wall friction coefficient for the diffuser was determined based on measurement of the pressure drop between the upstream and downstream ends, and the corresponding flow velocity in the diffuser.

The measurement of the contraction coefficient C_d for the open ports was obtained using Eq. (5f). In order to evaluate the two controlling parameters, α and β , in different ranges, three different port openings (i.e., 6, 12 and 20 sequential ports counting from the upstream end) were selected. As shown in Fig. 2, flow measurement at all the open ports was done using graduated polypropylene beakers (1000-ml capacity) beneath the discharging ports and a stop watch (error range = 0.01 sec). To ensure accuracy of the data, every flow rate measurement was repeated three times, and the average value was adopted (Cheng, 1997).

IV. Results and Comparisons

1. Comparison between Theoretical and Numerical Results

As described by Eq. (9), the port discharge increases downstream in the case where the port momentum parameter, β , is dominant ($\alpha = 0$). The results obtained using the firstorder perturbation method (i.e., Eq. (18)) and numerical simulations in the range of $0 \le \alpha \le 0.5$ are shown in Fig. 3. The uniformity parameter, V_r , shown as the contour lines in Fig. 3, is the same when obtained using both methods when $\alpha =$ 0, and there is little when α is in the range of $0 \le \alpha \le 0.2$. As shown in Fig. 3, when $\beta \ge \alpha$, V_r decreases as α increases if β remains a constant. Thus if the port momentum parameter is dominant, then the port discharge becomes more uniformly distributed when the wall friction parameter increases. In Fig. 3, the difference in V_r when obtained using the perturbation method and numerical simulation is obvious when the values of α and β are finite and are about the same order, which reflects



Fig. 3. The contours of the uniformity index Vr for the port discharge distribution based on results obtained using the perturbation method (a) and numerical simulation (b).



Fig. 4. The contours of the uniformity index *Vr* for the port discharge distribution based on the results obtained using (a) Shen's solution and (b) numerical simulation.

the limitation of the perturbation method.

In order to evaluate the uniformity parameter, V_r , in further detail, numerical simulations were carried out under a wider range for α and β . The results are shown in Fig. 4(b) while the results obtained using Shen's solution (i.e., Eqs. (6) and (7a) – (7e)) are also plotted in Fig. 4(a) for comparison. As mentioned in Section II, Shen's solution is incorrect since it does not satisfy the governing equation (Cheng, 1997). As shown in Fig. 4(b), the contour line of the uniformity parameter, V_r , equaling unity can be obtained under the condition $\beta =$ 0.594 α . According to Eqs. (5b) and (5c), the corresponding diameter of the diffuser that maintains uniform port discharge is as follows:

$$D_h = \frac{0.353}{2 - \gamma_d} fL . \tag{25}$$

If the diameter *D* is greater than D_h , then the flow distribution for the ports will increase downstream. On the other hand, if $D < D_h$, the outflow will decrease downstream.

2. Comparison between Experimental and Numerical Results

J. Sherman's experimental data¹ were used to verify the theoretical and numerical results. The diffuser used in Sherman's

experiment was 152.4 cm long, the diffuser diameter =27.3 cm, the port diameter = 8.64 cm, f = 0.014, Ar = 2.0, $\alpha = 0.203$ and $\beta = 0.747$. As shown in Fig. 5, the numerical results agree well with the experimental results. The analytical solution based on Eq. (18) is also shown in Fig. 6, and its deviation from the experimental results is mainly attributed to its simplified approximation where α is much less than β .

Figures 6 - 8 show the port discharge distribution for 6,12 and 20 port openings, respectively, from the experimental data in the present study. The experimental parameters for 6-port openings, shown in Fig. 6, are Ar = 0.236, $C_d = 0.629$, $\alpha = 0.203$ and $\beta = 0.334$. Similarly, the parameters for the 12-port openings shown in Fig. 7, are Ar = 0.469, $C_d =$ 0.625, $\alpha = 0.524$ and $\beta = 0.663$, and they are Ar = 0.75, C_d = 0.60, α = 1.01, and β = 1.06 for the 20-port openings shown in Fig. 8. As shown in Figs. 6, 7 and 8, the numerical results agree well with the experimental data while those obtained using Shen's solution (Eq. (6)) are obviously different from the experimental data. According to Eq. (25), $D_h = 0.87$ cm for the experimental conditions shown in Figs. 6-8. Since the diameter of the diffuser (D = 2 cm) is greater than D_h , the port discharge increases downstream. The average port discharges shown in Figs. 6 – 8 are all around 70 cm³/s, and α and β are of the same order. As the port number increases,



Fig. 5. The port discharge distribution of J. Sherman's experimental data compared with numerical and analytical results.



Fig. 6. The port discharge distribution of the experimental data and numerical simulation (6 ports, Re = 27000).

¹ Sherman, J. (1949) Internal Report, Research and Development Center, The Babcock and Wilcox Company, Alliance, OH, U.S.A. (not available in the open literature but the experimental data were cited in Bajura's paper (Bajura, 1971)).



Fig. 7. The port discharge distribution of the experimental data and numerical simulation (12 ports, Re = 53363).



Fig. 8. The port discharge distribution of the experimental data and numerical simulation (20 ports, Re = 91660).

the values of Ar, α and β all increase accordingly. As shown in Fig. 4(b), the uniformity parameter, V_r , deviates more obviously from unity when the values of β/α are farther away from 0.594, and the deviation grows with increasing values of α and β . Consequently, the measured port discharge distribution shown in Fig. 8 is more uneven than that shown in Fig. 6. The increase of the port numbers thus causes outflow along the diffuser away from uniform distribution unless the diameter of the diffuser is close to D_h .

V. Conclusions

The controlling parameters for the port discharge distribution along a multiple-port diffuser are the wall friction parameter, *α*, and the port momentum parameter, *β*. The analytical solutions for the outflow distribution were derived in this study, i.e., Eqs. (18) and (24), under the condition that either *α* or *β* is the dominant parameter. As the value of *α* is negligible, i.e., *β* is dominant, such as for a short diffuser, the port discharge increases downstream with a maximum value at the downstream end. On the other hand, if *α* is the dominant parameter, *β* is the dominant parameter, and the port discharge increases downstream with a maximum value at the downstream end.

such as for a long diffuser, then the port discharge decreases downstream. Consequently, the maximum port discharge occurs at the upstream end.

(2) Evaluation of the combined effects of α and β was performed in this study by means of numerical simulation. The results agree well with both analytical solutions and experimental data. In order to make the port discharge even, the parameters α and β should follow the relationship of $\beta \approx 0.594\alpha$, or the diameter of the diffuser, *D*, should be close to the value of $D_h = \frac{0.353}{2 - \gamma_d} fL$.

The port discharge increases downstream if $D > D_h$, and the port discharge decreases downstream if $D < D_h$.

(3) The increase in port numbers for a diffuser will simultaneously cause the values of both α and β to increase. Consequently, the port outflow under the condition of $\beta \neq 0.594\alpha$ will deviate from a uniform distribution as the port openings increase, according to the results of numerical simulations as well as experimental data.

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Outflow along Multi-port Diffusers

多孔擴散管孔口流量分布之分析

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摘要

本文以理論及數値解析探討多孔擴散管之孔口流量分佈,並進行實驗以驗證理論解及數値解之正確性。影響多 孔擴散管之孔口流量分佈之主要物理因子可歸納為管壁摩擦損失係數 α 及孔口動量係數 β 兩個無因次參數。當管壁 摩擦損失係數可忽略時,流量分佈隨孔口動量係數之增加而趨於不均匀,且沿下游遞增以尾端之孔口流量最大;而當 管壁摩擦損失係數為主控因子時,則孔口流量以前端之孔口流量最大,並沿下游呈遞減分佈。本文並以數値分析 $\alpha < \beta$ 二參數對孔口流量分佈之綜合影響,當此二個參數之比在一定範圍時,即 $\beta = 0.594\alpha$,或多孔擴散管之管徑應接近 0.353fL/(2- γ_0),方能使各孔口流量分佈趨於均匀。