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# A Regression Relation between Cavitation Number and Cavity Length for Two-Dimensional Supercavitating Hydrofoils

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## ABSTRACT

For convenience in engineering applications, we attempt to find a general regression relation between the nondimensional cavity length and the cavitation number for two-dimensional supercavitating hydrofoils. Based on observations of the general trend of the relation, a nonlinear function for these two physical quantities is proposed, and the coefficients in the function are optimally determined via a least-squares procedure and an iterative Gauss-Newton method. Several supercavitating hydrofoils, including flat-plate hydrofoils and hydrofoils with and without cambers, are investigated. The results seem to indicate that there exists a global approximate relation, the optimum coefficients of which depend on the flow conditions and hydrofoil shape.

**Key Words:** 2-D supercavitating hydrofoil, potential flow, regression relation

## I. Introduction

Although the details of flow physics are quite complicated and not clearly understood even today, the macroscopic approach for cavitating or free streamline flows has been significantly improved and successfully applied in engineering analyses in the past few decades. Progress has been made in terms of theoretical and computational advances.

Theoretically, the macroscopic features of either partially cavitating or supercavitating flow have been successfully modeled using the potential flow theory (e.g., Wu (1969, 1972)). In this approach, there are several typical features. For example, the detailed flow structure is usually neglected, and phenomena such as the two-phase and turbulent characteristic are ignored. In addition, the cavitating region is ideally treated as a uniform gas film without liquid in it, and the flow outside the cavitating region and the body is assumed to be inviscid, irrotational, and incompressible.

In the context of potential flow, one needs to specify either a cavitation number or a cavity length to complete the theoretical formulation of the prob-

lem. This specification implies that either the cavitation number or cavity length must be functionally expressed in terms of the other. Generally speaking, such a relation cannot be expressed explicitly in terms of elementary functions. However, finding the relation between these two physical quantities has engineering significance in modern computational cavitating flow mechanics. This point will be further addressed later. In the following, we will first give a brief overview of the literature on the search so far for such a relation.

Following the linear supercavitating-flow theory first introduced by Tulin (1953, 1955, 1964), Geurst (1960) employed a conformal transformation technique to derive an implicit theoretical relation among the angle of attack, cavitation number, and nondimensional cavity length (*i.e.* the ratio of the cavity length to the chord of the hydrofoil) for a supercavitating flow. The relation contains coefficients in integral form which can be evaluated using a typical numerical quadrature procedure. Nevertheless, the cavity length can be readily obtained for a given cavitation number, or vice versa, provided that the incident angle of uniform flow is specified. Further-

more, he also studied the special case of a flat-plate hydrofoil and obtained a simple analytical expression for these physical quantities. In addition, Acosta (1955) provided the first partial cavitation solution specifically for a flat plate hydrofoil. He also expressed the cavitation number explicitly in terms of the nondimensional cavity length and the angle of attack. These seem to be the only analytical relations available at the present. Of course, all these results is applicable only to flow at a small angle of attack, as the assumptions of the linear theory imply.

With progress in the development of computational methods, nonlinear analyses of cavitating flows have been conducted using various numerical methods. Brennen (1969) employed a finite-difference method to study axisymmetric flows. Later on, several potential-based or velocity-based boundary element methods were developed and applied, including those of Pellone and Rowe (1981), Uhlman (1989), Lee *et al.* (1992), and Kinnas and Fine (1993). All these developments enhance our understanding of cavitating flow and our ability of foil design involving cavitating flow phenomena.

However, this progress has not benefited our understanding of the relation between the cavitation number and cavity length. On the contrary, the fact is that there is even a dilemma in computations for design purposes. From the computational point of view, prescribing a cavity length is most natural since, otherwise, any discretization is impossible, let alone further computations. In fact, most of the important contributions made to computational development of cavitating flow lie in the category which expresses the cavitation number as a function of cavity length (e.g., Lee *et al.* (1992) and Uhlman (1989)). Nevertheless, from the engineering point of view, such a computational approach is not only unnatural, but also quite indirect because the cavitation number, rather than the cavity length, is usually specified as a design criterion; therefore, its value, instead of the cavity length, is often assigned. Consequently, it would be convenient in a computational design-analysis process if the relation between the nondimensional cavity length and cavitation number were available for a given angle of attack.

Little work has been devoted to understanding the relation between the cavitation number and cavity length. This relation has seldom been investigated directly. Instead, several methods that employ concepts of inverse problem have been proposed in the literature. The feature of this approach is that the problem must be linearized. Davies (1970) first formulated the linearized cavitating hydrofoil problem in terms of singular integral equations with respect

to unknown vorticity and source distributions, and inverted the resulting integral equations. Unfortunately, the final expressions for the vorticity and source distributions were coupled to each other. Persson (1978) focused on a supercavitating flat plate and inverted the equations to obtain analytical expressions for their distributions. Later, Kinnas (1992) extended the analytical inversion to supercavitating hydrofoils of arbitrary shape. He expressed the cavitation number, the vorticity and source distributions in terms of integrals of quantities which depended only on the foil geometry and the cavity length. Since this approach is linear, the inherent deviation of the numerical solution from the exact one becomes non-negligible when the angle of attack is not small and the cavity bubble is thick.

In the present study, we will restrict our focus to supercavitating flow and attempt to find a general relation between the nondimensional cavity length and cavitation number for two-dimensional hydrofoils through a regression analysis procedure. The analysis procedure is comprised of several steps. First of all, assuming the flow is inviscid and irrotational, we compute the flow past a supercavitating hydrofoil for various cavity lengths to find the corresponding cavitation numbers. We conduct the computation using a nonlinear model; that is, the shape of the cavity is a part of the solution, and the boundary conditions on the cavity surface are specified on the exact surface. As a result, the results we obtain are valid for flows at any angle of attack. Then, we will proceed to examine here the trend of the relation between the cavity length and cavitation number and propose a nonlinear exponential polynomial to fit the relation via a least squares procedure. Finally, we will examine this fitting and propose an expression which may be globally valid for all two-dimensional supercavitating hydrofoils at any angle of attack.

In the following, we will first briefly introduce the theoretical governing equations and boundary conditions and the boundary element method which we have employed to find the solution for a flow past a two-dimensional supercavitating hydrofoil. Then, based on careful observation of the trend of the relation between the nondimensional cavity length and the cavitation number, we will propose a nonlinear model to fit the relationship. Finally, three typical supercavitating hydrofoils will be examined. The three hydrofoils include one with a flat-plate cross section (without thickness and camber), one with a NACA 16-004 cross section (without camber but with thickness), and one with Kehr's new section (with thickness and camber) (Kehr, 1998). For the sake of conciseness, only the results of the three

hydrofoils will be presented in this report. However, all the conclusions given in the present report represent those drawn from these three hydrofoils and many other tests using supercavitating hydrofoils of different sections, including the NACA 16 series of different thickness and supercavitating sections.

## II. Governing Equation and Boundary Conditions

Figure 1 schematically depicts a uniform flow past a supercavitating hydrofoil. A closed cavity is formed with a surface  $S_c$ . Within the scope of potential flow, the flow past a supercavitating hydrofoil can be expressed in terms of the velocity potential,  $\Phi(x,y)$ , which is governed by the Laplace equation

$$\nabla^2 \Phi = 0. \quad (1)$$

The velocity of the flow field can be obtained by taking the gradient of  $\Phi$ ; that is,  $\mathbf{u} = \nabla \Phi$ .

In addition to the governing equation, there exist several boundary conditions for the supercavitating flow. They are identified as follows.

- (1) *Undisturbed flow at far field.* Far away from the hydrofoil, a uniform distribution is assumed:

$$\nabla \Phi = U(i \cos \alpha + j \sin \alpha), \quad (2)$$

where  $\alpha$  is the angle of attack of the uniform flow with a magnitude of  $U$ .

- (2) *Kinematic condition.* On the hydrofoil and cavity surfaces, the flow is tangent to the surface:

$$\nabla \Phi \cdot \mathbf{n} = 0, \quad (3)$$

where  $\mathbf{n}$  is the outward unit normal to the surfaces.

- (3) *Dynamic condition.* The pressure inside the cavity is constant. That is, on the cavity sur-

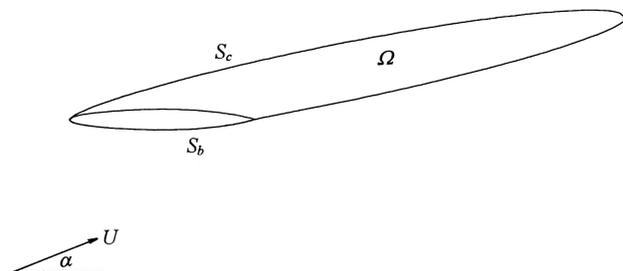


Fig. 1. Schematic diagram of the flow field.

face,  $S_c$ ,

$$p = p_v, \quad (4)$$

where  $p_v$  is the vapor pressure. The pressure on the cavity and foil surfaces is related to the local tangential speed by the Bernoulli equation, from which we can obtain the pressure coefficient,

$$C_p = \frac{p - P}{\frac{1}{2} \rho U^2} = 1 - \left( \frac{|\mathbf{u}_t|}{U} \right)^2 \quad (5)$$

and the cavitation number

$$\sigma \equiv \frac{P - p_v}{\frac{1}{2} \rho U^2} = -C_{pv} = \left( \frac{|\mathbf{u}_c|}{U} \right)^2 - 1, \quad (6)$$

where  $P$  is the undisturbed pressure,  $\rho$  the density of the fluid,  $\mathbf{u}_t$  the tangential velocity on the boundary and  $\mathbf{u}_c$  the tangential velocity on the cavity surface. Since the pressure inside the cavity is constant, we may equivalently designate the value of the tangential velocity or the cavitation number as the dynamic condition.

- (4) *Kutta condition.* In the present supercavitating flow, the condition should be applied at the end of the cavity, rather than at the trailing edge of the hydrofoil.
- (5) *Detachment condition.* Generally speaking, such a specification is beyond the scope of potential flow theory. Here, we simply assume in the first stage that the flow detaches exactly at the leading and trailing edges and adjust the positions of the detachment points if the solution thus obtained shows non-physical behaviors. This condition also requires that the velocity distribution be continuous at the detachment point of the trailing edge; that is,

$$\lim_{A \rightarrow T.E.D.} \mathbf{u}_A = \lim_{B \rightarrow T.E.D.} \mathbf{u}_B, \quad (7)$$

where points  $A$  and  $B$  represents points on the hydrofoil and cavity surfaces, respectively; *T.E.D.* is the abbreviation for the trailing edge detachment point.

- (6) *Termination condition of the cavity.* A termination model must be applied at the end of the cavity. Here, we employ for simplicity a closed-cavity model. Mathematically, this condition can be expressed as

$$T^c(x_{cep}) = 0, \quad (8)$$

where  $T^c(x)$  represents the thickness of the cavity at the ordinate  $x$  and  $x_{cep}$  the cavity end point.

With the set of governing equation and boundary conditions, we can find numerically the cavity surface, the flow field, and the cavitation number through iterations, provided that a cavity length is specified. The numerical method we employ in the present study is briefly described here. For detailed discussion, see Chen and Weng (1999a).

The particular numerical method we use is a low-order boundary element method. A dipole distribution is introduced on the cavity and foil surfaces. In addition, since the boundary of the cavity is unknown, a source distribution must be introduced on the cavity surface. The distribution of sources on the cavity surface serves as a normal flux generator so as to form the cavity surface. Because of its thickness-forming function, the cavity surface may be adjusted according to this distribution. The following iterative procedure proposed by Lee *et al.* (1992) can be employed to find the solution.

- (1) Prescribe the length of the cavity surface and an initial shape of the cavity surface.
- (2) Determine the dipole and source strengths. Use an LU decomposition procedure in the linear algebraic solution scheme.
- (3) Update the shape of the cavity surface based on the source distribution obtained in the previous step.

Repeat steps (2) and (3) till reasonable convergence is achieved.

### III. A Nonlinear Model

As discussed in the previous section, the cavity length can be expressed as a function of the cavitation number, provided that the shape of the hydrofoil and the inflow angle of attack are given. To secure a proper curve fitting, we first observe the general trend of the relation which relates the nondimensional cavity length to the cavitation number. Generally speaking, for a given hydrofoil and flow angle of attack, the trend indicates that a longer cavity corresponds to a smaller cavitation number in a nonlinear manner, as in the typical case shown in Fig. 2.

To explicitly fit the implicit relation between the cavity length and cavitation number, we propose for a given angle of attack a function of the following form:

$$\begin{aligned} \sigma &= f(l, b_1, b_2, b_3, b_4, b_5) \\ &= b_1 \exp(b_2 l / c) + b_3 \exp(b_4 l / c) + b_5, \end{aligned} \quad (9)$$

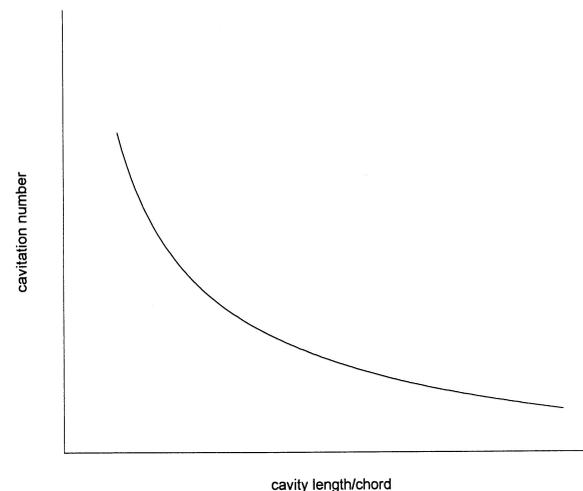


Fig. 2. A typical relation between the cavity length and the cavitation number.

where  $l$  is the cavity length and  $b_i$  ( $i = 1, 2, \dots, 5$ ) are some unknown coefficients. The reasons why we propose such a functional relation are two-fold. First, as mentioned above, the typical trend of these two variables shown in Fig. 2 indicates that such a composition of exponential functions is proper. Second, the analytical relation for a flat-plate hydrofoil can be expressed locally in terms of a series of exponential functions. Therefore, it seems feasible to employ the expression of finite terms, Eq. (9), to approximate the relation and to extend its applicability to hydrofoils with an arbitrary shape of section.

In the following, we will examine the feasibility of this approach. This examination will consist of two steps. In the first step, we will study the possibility of approximating the relation using Eq. (9). This will be examined in this section. In the second step, we will attempt to simplify Eq. (9) to achieve a global relation with fewer parameters in order to make engineering application more convenient. This will be discussed in the next section.

Now, we will proceed to investigate the possibility of using Eq. (9) to represent the relation between the nondimensional cavity length and the cavitation number. Our remaining task is to find the unknown coefficients which best fit the relation. This can be accomplished by employing the Gauss-Newton method. The procedure is briefly described as follows. Given a set of initial guesses  $(b_1^{(0)}, b_2^{(0)}, b_3^{(0)}, b_4^{(0)}, b_5^{(0)})$ , assume that their deviations from the optimum values  $(b_1, b_2, b_3, b_4, b_5)$  are  $(\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5)$ . Then, we have

$$b_i = b_i^{(0)} + \Delta_i, \quad i = 1, 2, \dots, 5.$$

To find the values of  $\Delta_i$  iteratively, Eq. (9) is linearized:

$$\sigma = f(l, b_1, b_2, b_3, b_4, b_5) = f_{k0} + \sum_{i=1}^5 \frac{\partial f_{k0}}{\partial b_i} \Delta_i, \quad (10)$$

where

$$f_{k0} = f(l, b_1^{(0)}, b_2^{(0)}, b_3^{(0)}, b_4^{(0)}, b_5^{(0)})$$

$$\frac{\partial f_{k0}}{\partial b_i} = \frac{\partial f(l, b_1^{(0)}, b_2^{(0)}, b_3^{(0)}, b_4^{(0)}, b_5^{(0)})}{\partial b_i}.$$

Then, we can employ the least-squares concept to determine these five unknown constants. That is, we required that

$$Q = \sum_{k=1}^N [\sigma_k - f(l, b_1, b_2, b_3, b_4, b_5)]^2 \quad (11)$$

be a minimum, where  $N$  is the total number of data points. This condition leads to the following simultaneous linear equation system:

$$\sum_{j=1}^5 a_{ij} \Delta_j = a_{iy}, \quad i = 1, 2, \dots, 5, \quad (12)$$

where

$$a_{ij} = \sum_{k=1}^N \frac{\partial f_{k0}}{\partial b_i} \frac{\partial f_{k0}}{\partial b_j}, \quad i, j = 1, 2, \dots, 5,$$

$$a_{iy} = \sum_{k=1}^N \frac{\partial f_{k0}}{\partial b_i} (\sigma_k - f_{k0}), \quad i = 1, 2, \dots, 5.$$

Solving the system in Eq. (12), we can then obtain the deviation  $\Delta_i$ , which enables us to iterate the procedure till proper convergence is achieved.

Some case studies were carried out to examine the feasibility of the curve fitting using Eq. (9). First of all, we studied the supereavitating flat-plate hydrofoil. The results are shown in Fig. 3. For different angles of attack and cavity lengths, we did find that there always existed a proper set of coefficients  $b_i$  ( $i = 1, 2, \dots, 5$ ) which could be used to achieve a remarkable fitting. Their values for each case are shown in Table 1. In each curve fitting, we have  $L_2 < 10^{-4}$ , where  $L_2$  is the average residue, defined as

$$L_2 = \sqrt{\frac{Q}{N}}.$$

It is significant that properly selecting the values of

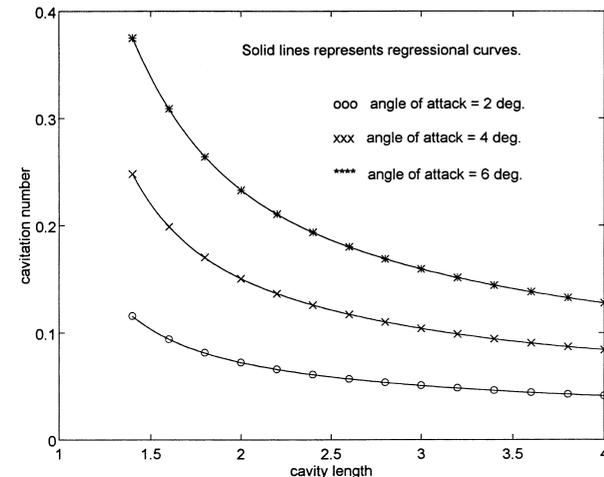


Fig. 3. Curve fitting of a flat-plate hydrofoil.

Table 1. Optimum Coefficients for a Flat-Plate Hydrofoil

	$\alpha = 2^\circ$	$\alpha = 4^\circ$	$\alpha = 6^\circ$
$b_1$	0.1472	0.3353	0.3531
$b_3$	4.6747	13.0427	4.8961
$b_5$	0.0317	0.0652	0.0835
$b_2$	-0.68	-0.72	-0.52
$b_4$	-3.68	-3.84	-2.64

parameters in Eq. (9) enabled us to approximate the exact relation quite well.

We then proceeded to a real hydrofoil with an NACA16-004 section. The results are shown in Fig. 4 and Table 2. Again, for each of fitting, we have  $L_2 < 10^{-4}$ . Obviously, the function fits the computed data quite well.

Finally, we also conducted some tests for a hydrofoil with a cambered cross section which was

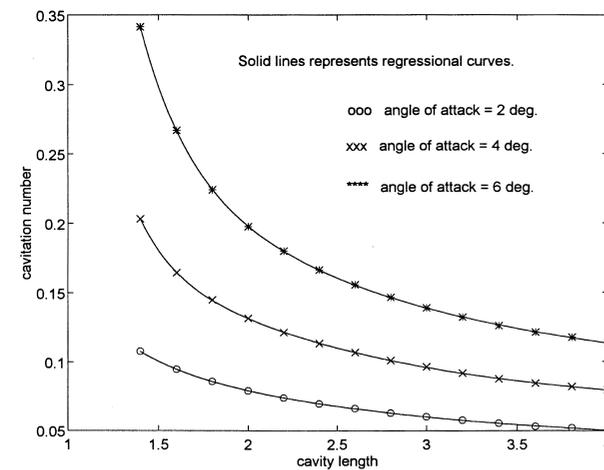


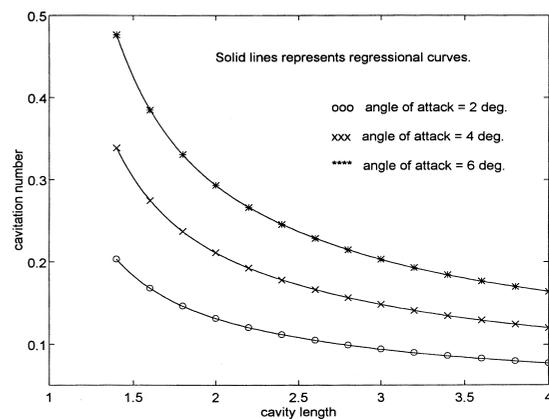
Fig. 4. Curve fitting for a hydrofoil with an NACA16-004 section.

**Table 2.** Optimum Coefficients for an Hydrofoil with an NACA 16-004 Section

	$\alpha = 2^\circ$	$\alpha = 4^\circ$	$\alpha = 6^\circ$
$b_1$	0.1328	0.2859	0.3785
$b_3$	1.8420	77.1770	23.3730
$b_5$	0.0393	0.0648	0.0863
$b_2$	-0.62	-0.74	-0.66
$b_4$	-3.56	-5.46	-3.86

designed by Kehr (1998). Similar to the previous two cases, the function in Eq. (9) gives a satisfactory result, as shown in Fig. 5 and Table 3. For each fitting, we still have  $L_2 < 10^{-4}$ .

From the three tests described above, we can draw some conclusions. First of all, for the three different hydrofoils, we observe that Eq. (9) can be used to describe as accurately as possible the local relation of the cavitation number to the nondimensional cavity length, provided that the values of the parameters are prescribed properly. This observation is valid for various angles of attack of the incoming flow. In fact, the three test cases represent three typical categories of cross sections of hydrofoils. In addition to these three typical tests, we also carried out many other tests using different hydrofoils which all fall within these three categories, and the same

**Fig. 5.** Curve fitting for a hydrofoil with a new section (Kehr, 1998).**Table 3.** Optimum Coefficients for a Hydrofoil with a New Section

	$\alpha = 2^\circ$	$\alpha = 4^\circ$	$\alpha = 6^\circ$
$b_1$	0.2603	0.4469	0.6500
$b_3$	8.0396	15.8194	23.4738
$b_5$	0.0600	0.0931	0.1277
$b_2$	-0.68	-0.70	-0.72
$b_4$	-3.74	-3.80	-3.82

conclusion was reached. Therefore, we may be able to conclude that Eq. (9) can properly describe the local relation between the cavity length and the cavitation number.

Secondly, according to the computed results shown in Tables 1 to 3, it seems that the values of the parameters for the best fitting strongly depend on both the shape of the hydrofoil and the inflow angle of attack. This is especially true for the coefficient parameters of each term,  $b_1$ ,  $b_3$  and  $b_5$ . Unfortunately, it seems that the rule governing the variation of each parameter has different trends for different flow conditions and hydrofoil shapes and cannot be explicitly and clearly elucidated. Meanwhile, it is also observed that the variations of the exponent parameters  $b_2$  and  $b_4$  for different hydrofoil cross sections and at different inflow angles of attack are less significant, especially compared to those of the coefficient parameters. This interesting feature leads us to the second step, in which we attempt to simplify Eq. (9) in order to achieve a somewhat relaxed relation which, however, contains fewer parameters.

#### IV. A Somewhat Relaxed Relation with Fewer Parameters

As discussed in the previous section, the model in Eq. (9) with appropriate coefficients can indeed well fit the relation between the cavitation number and the nondimensional cavity length for various flow conditions and hydrofoil shapes. According to the conclusions given in the last section, a question that can naturally arise is whether a somewhat relaxed relation with fewer parameters can be sought for various two-dimensional supercavitating hydrofoils. Of course, such a relation should still be accurate enough to have engineering significance and be applicable; however, it should also be relaxed “enough” to accommodate individual characteristics of various hydrofoils.

As revealed by the test cases studied in Section III, the parameters in Eq. (9) vary quite significantly from one flow condition or one particular hydrofoil to another if we attempt to obtain the best fitting for the relation of the cavity length to the cavitation number. Therefore, to achieve a relaxed relation with fewer parameters, we must somewhat relax the value of  $L_2$ , which represents in a general sense the accuracy of the curve fitting to the nonlinear relation. In other words, rather than require that it be as small as possible, we have to keep the average least-squares residue error  $L_2$  at some tolerable value without loss of engineering significance, say  $10^{-3}$ .

Such a relaxation certainly leads to an interesting fact that for a given value of  $L_2$ , there exist many sets of  $(b_1, b_2, b_3, b_4, b_5)$  which meet the required error tolerance condition. Generally speaking, a larger  $L_2$  gives rise a broader allowable range for these parameters.

Since there are many choices for  $(b_1, b_2, b_3, b_4, b_5)$  under a relaxed accuracy requirement, we may be able to reduce the number of parameters. Therefore, we will investigate whether there exists a proper fixed set of exponent parameters ( $b_2$  and  $b_4$ ) for various supercavitating hydrofoils to get a least-squares fitting for which the deviation from the theoretical (but computed) relation in a general sense is less than the prescribed value of  $L_2$ . We choose to fix the exponent parameters, rather than the coefficient parameters, because, as observed in the previous section, their variations are less significant for different hydrofoils and incoming flow angles of attack. Such an additional requirement reduces the number of unknown coefficients.

The remaining task is to examine the best fit of Eq. (9) to the computed data in a least-squares sense, given the values of  $b_2$  and  $b_4$ . In the following, the computational results will be presented.

For a flat-plate hydrofoil with  $\alpha = 2^\circ, 4^\circ$ , and  $6^\circ$ , Fig. 6 show the level curves of constant values of  $L_2$  for the best fit at different values of  $b_2$  and  $b_4$ . It is obvious that, from the plots, there exists a common region within which the value of  $Q$  for best fit is less than  $10^{-3}$ .

Now we can proceed to examine hydrofoils with an NACA16-004 section and a new section. The results are shown in Figs. 7 and 8, respectively. The same conclusion can be drawn.

Some conclusions can be drawn based on observations of these plots. First of all, the level curve trends seem to not be strongly dependent on the incident angle of flow or shape of the hydrofoil section. All the sets of level curves are similar to one another though those regions within which the average residue error is less than some given value do differ. In fact, tests conducted using other hydrofoils also showed a similar trend. Second, observing the results for the present three supercavitating hydrofoils, we find that there exists a common region defined by  $(b_2, b_4)$  within which the average residue errors are less than some level of tolerance. In fact, it is obvious that for a larger level of tolerance, the common region becomes even broader. This, in turn, implies that we may choose, in all cases, common values of  $b_2$  and  $b_4$  such that there always exists for different hydrofoils and incoming flow conditions an optimum set of  $(b_1, b_3, b_5)$  which

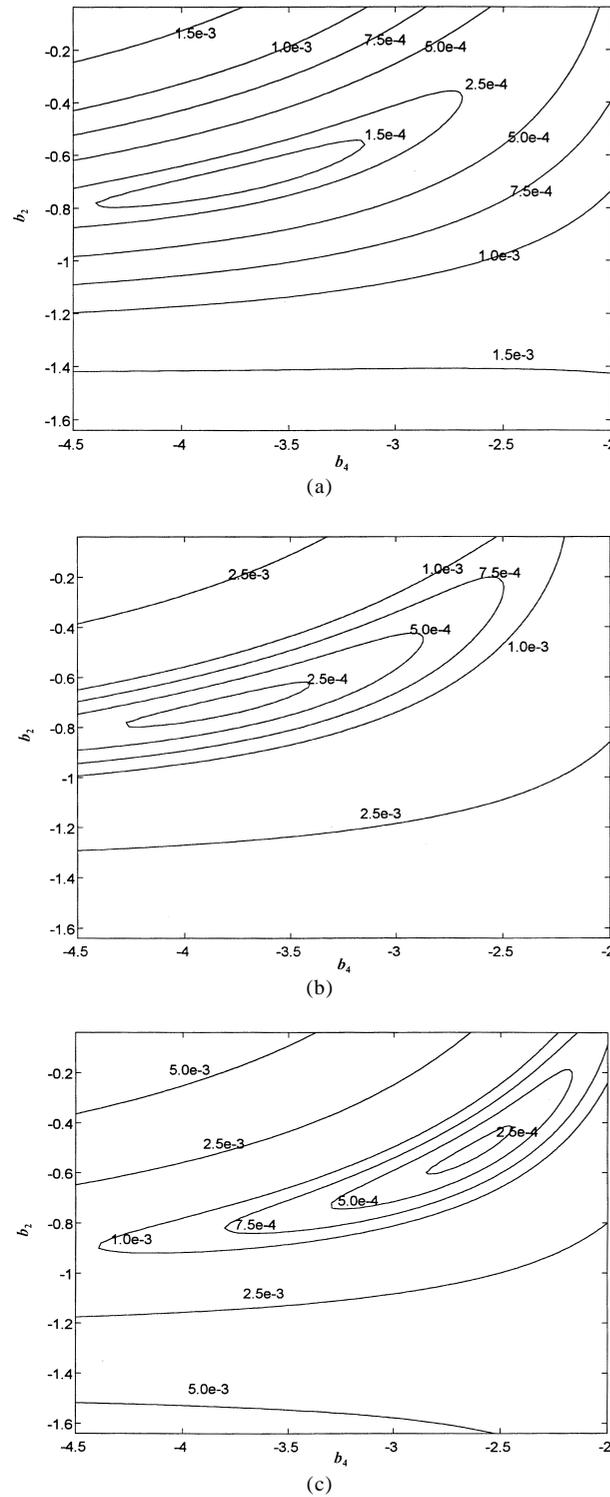
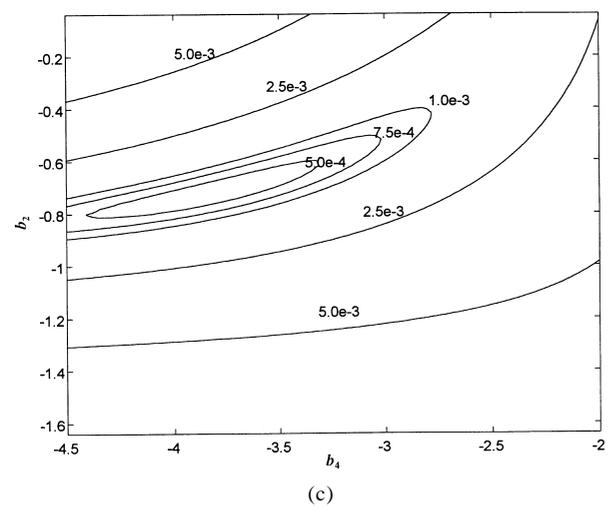
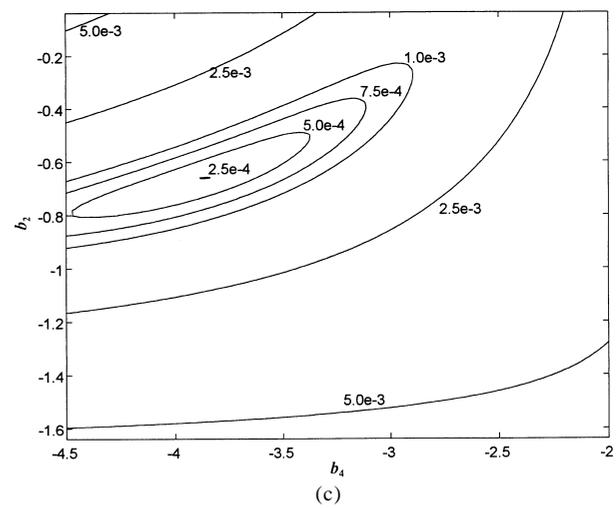
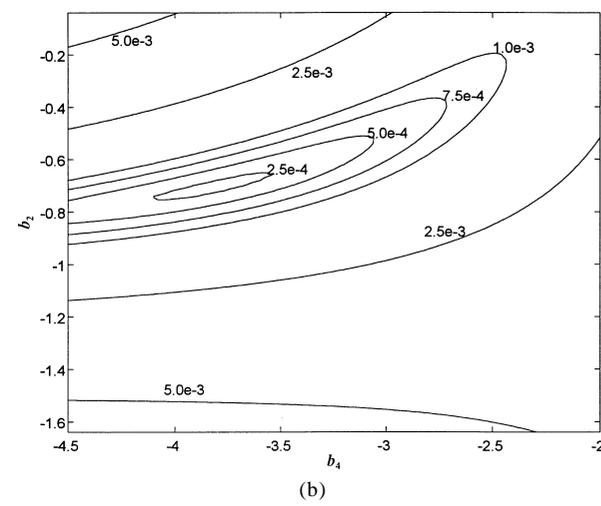
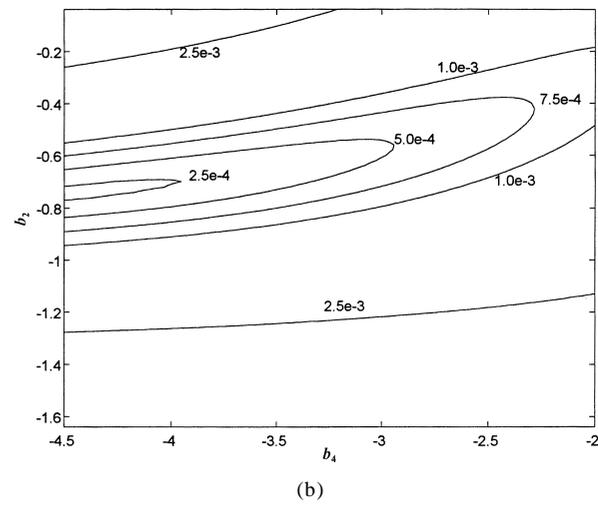
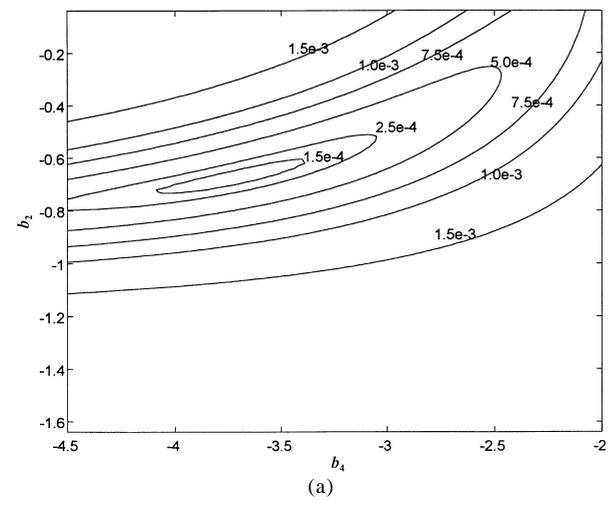
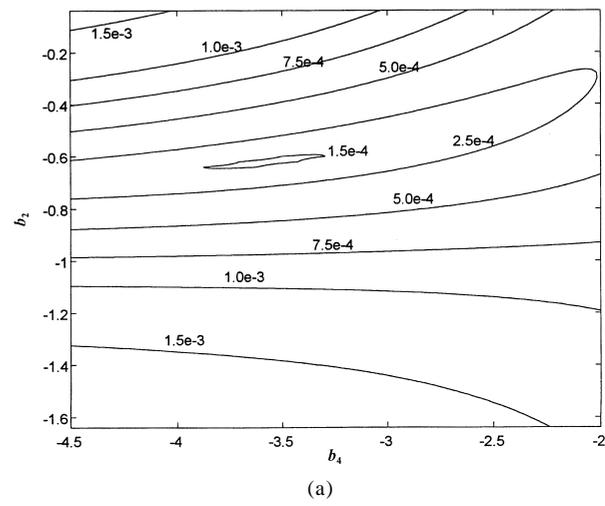


Fig. 6. Level curves of  $L_2$  for a flat-plate hydrofoil at (a)  $\alpha = 2^\circ$ , (b)  $\alpha = 4^\circ$  and (c)  $\alpha = 6^\circ$ .

keeps the fitting within the prescribed level of tolerance. For example, we can choose  $b_2 = -0.7$  and  $b_4 = -3.0$ . Then, we have a somewhat relaxed but still

### Regression Relation for 2-D Hydrofoils



**Fig. 7.** Level curves of  $L_2$  for a hydrofoil with NACA16-004 section at (a)  $\alpha = 2^\circ$ , (b)  $\alpha = 4^\circ$  and (c)  $\alpha = 6^\circ$ .

**Fig. 8.** Level curves of  $L_2$  for a hydrofoil with a new section at (a)  $\alpha = 2^\circ$ , (b)  $\alpha = 4^\circ$  and (c)  $\alpha = 6^\circ$ .

quite accurate relation with only three parameters:

$$\begin{aligned}\sigma &\approx f(l, b_1, b_3, b_5) \\ &= b_1 \exp(-0.7l/c) + b_3 \exp(-3.0l/c) + b_5.\end{aligned}\quad (13)$$

## V. An Application

Finally, we will present an application of the regression relation. Given a cavitation number, flow conditions, and a hydrofoil, we conducted tests to examine the feasibility of applying the regression relation to analyze flow past a supercavitating hydrofoil. Since there are three parameters in Eq. (13), we need to compute three flow solutions which correspond to three different cavity lengths. The cavitation numbers corresponding to the three cavity length can be found from the flow solution. Then, we can obtain a good approximate relation for further determination of the exact flow solution at the prescribed cavitation number. The details of the procedure can be found in Chen and Weng (1999b).

The first test case was the simplest flat-plate hydrofoil. The test conditions and results are shown in Table 4. The three prescribed nondimensional cavity lengths were 1.8, 3.0, and 3.2, respectively. Through computations, the corresponding computed cavitation numbers were found to be 0.1703, 0.1041, and 0.0989. Using these data, we calculated the coefficients of Eq. (13). Then, in two more iterations, we obtained the solution for which the cavitation number was accurate up to the fourth decimal point. Even at the first iteration, the relative error of the computed cavitation number compared to the prescribed value was less than one percent, and the relative error of lift coefficient was even smaller.

In the second test case, we studied a hydrofoil with NACA 16-004 section which had thickness but no camber. The test conditions were set at  $\alpha = 4^\circ$  and  $\sigma = 0.15$ . The various prescribed and computed values are listed in Table 5. This test case was somewhat special and complicated in that face cavitation was observed. In fact, different cavitation numbers (or, equivalently, different nondimensional

**Table 4.** Iteration Data for a Flat-Plate Hydrofoil at  $\alpha = 4^\circ$  and  $\sigma = 0.15$

Flat Plate ( $\alpha = 4^\circ$ , $\sigma = 0.15$ )					
No.	Initial Guess			Iterations	
	1	2	3	1	2
$l/c$	1.8	3.0	3.2	2.0039	2.0083
$\sigma$	0.1703	0.1041	0.0989	0.1506	0.1502
$C_L$	0.2149	0.1542	0.1501	0.1953	0.1945

**Table 5.** Iteration Data for a Hydrofoil with an NACA16-004 Section at  $\alpha = 4^\circ$  and  $\sigma = 0.15$

NACA16-004 ( $\alpha = 4^\circ$ , $\sigma = 0.15$ )					
No.	Initial Guess			Iterations	
	1	2	3	1	2
$l/c$	1.4	2.0	3.0	1.7697	1.7570
$\sigma$	0.2033	0.1314	0.0962	0.1473	0.1486
$C_L$	0.1483	0.1203	0.0999	0.1295	0.1305

**Table 6.** Iteration Data for a Hydrofoil with a New Section at  $\alpha = 6^\circ$  and  $\sigma = 0.3$

New Section by Kehr ( $\alpha = 6^\circ$ , $\sigma = 0.3$ )				
No.	Initial Guess			Iterations
	1	2	3	1
$l/c$	1.6	2.2	2.8	1.9618
$\sigma$	0.3847	0.2662	0.2149	0.2994
$C_L$	0.5207	0.4016	0.3554	0.4334

cavity lengths) led to different starting points on the face side beyond which face cavitation appeared. Nevertheless, the computed results indicated that convergence could be achieved within a few iterations.

Finally, we also conducted a study on a hydrofoil with a new section designed by Kehr (1998). This represents a hydrofoil having thickness and camber. Table 6 tabulates the test conditions and iterative results. In one iteration, the solution converged within one percent of error.

According to the results of the series of tests described above, it appears that the regression relation proposed in the present study is a convenient tool for analyzing supercavitating flow where a cavitation number is given. For any three arbitrary cavity lengths which serve as initial guesses, we can find within a few iterations the correct cavity length which corresponds to the given cavitation number; then, the supercavitating flow and the cavity can be readily found.

## VI. Concluding Remarks

In this study, we have discussed the regression relation between the cavity length and the cavitation number. We have found that the relation can be well fitted by a exponential polynomial with proper coefficients. Furthermore, within an acceptable range of accuracy, a relaxed relationship exists with constant exponents. This relation can be applied to semi-direct analysis of supercavitating flow when a cavitation number is given. Tests have been conducted, and we have found that this approach seems to be quite reliable and robust in terms of the number of iterations

and its applicability to hydrofoils with arbitrary sections differing in shape.

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## 二維全空化水翼之空化係數與空泡長度迴歸關係研究

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### 摘要

本文針對二維全空化水翼流場，探討空化係數與空泡長度間的迴歸關係式，以便工程上的應用。我們觀察此二者的一般關係趨勢，並提出合理的非線性函數關係式，其中的係數參數則藉由最小二乘方法與高斯-牛頓迭代法來決定。為了解此關係式的可行性，我們討論了各種不同類型的水翼，包括平維水翼、有厚度無拱高的水翼、以及有厚度有拱高的水翼。研究結果顯示，我們似可找到一個通用的近似關係式，而關係式中的最佳係數則是入流狀況與水翼翼型的函數；此外，本文也說明了此關係式的可能應用。