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Parametric Stability Analysis of Rotor-Bearing Systems

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ABSTRACT

A general method of analysis based on Liapunov's direct method is presented for studying the behavior of the nonlinear system of differential equations governing the motion of a rotor-bearing system in the neighborhood of its equilibrium point. A model comprised of an axially symmetric rigid appendage attached at an arbitrary location along a nonuniform spinning shaft mounted on two dissimilar eight component end bearings is adopted to develop stability criteria involving different system parameters. The stability boundaries presented graphically in terms of system nondimensionalised parameters are typical examples of the types of design information available to engineers through the equations provided in this paper. Among the results reached in this paper are the demonstration of the roles played by (1) bearing mass, (2) appendage mass and dimensions, (3) bearing principal stiffness and damping coefficient, and (4) bearing cross-coupling stiffness and damping coefficients in affecting the nature of system whirl stability.

Key Words: Liapunov's direct method, whirl stability, rotor-bearing systems

I. Introduction

Rotor-bearing systems are assemblies widely used in aerospace and mechanical industries. Power machinery, such as compressors and turbomachines, usually transmits power by means of rotor bearing systems. In recent years, due to the design trend toward high spin rates to raise the operating efficiency, the resulting instability problems and lateral vibration of the system has become aggravated. Therefore, research in stability and dynamic response of rotor-bearing systems has prospered in the past few decades (Vance, 1988).

The prevalent approach adopted for stability analysis of rotor-bearing systems in most of the literature (El-Marhomy, 1994; Chang and Cheng, 1993; Kirk and Gunter, 1976; Chivens and Nelson, 1975; Iwatsubo and Tomita, 1973) is the traditional approach where the governing equations of motion are first transformed into an eigen-value problem. Then from the solution of the exponential growth (unstable) or decay (stable), the stability criteria are established based on the resulting eigen values (critical speeds) and their system parametric dependance. It is also found in the pertinent literature that some authors (Castelli and Elrod, 1964; Cheng and Trumpler, 1963; Rao, 1983, 1984) adopted the fluid dynamics approach where the rotor stability problem is mainly examined in terms of the characteristics of the fluid film bearings. The Routh-Hurwits criterion has also been used by several authors (Kirk and Gunter, 1976; Gunter, 1966; El-Marhomy, 1997) to study the stability of linearized rotor-bearing systems. However, for nonlinear systems and for certain limiting cases, this criterion cannot be applied, and the adoption of another stability criterion is required.

The Liapunov's direct method is a powerful tool for examining "infinitesimal stability" or "stability in the large" of linear and nonlinear dynamical systems. This method provides a significant advantage in that sufficient conditions for stability can be obtained without explicity solving the equations of motion, which are, in general, nonlinear and impossible to solve analytically. It has frequently been applied successfully in examining attitude stability of satellites and space mechanics problems. In the area of rotor dynamics, however, very few investigations (Gunter, 1966; Grobov and Kantimer, 1978; El-Marhomy and Schlack, 1991) are found in the literature that adopt this technique. Moreover, the first two focus mainly on the effect of appendage flexibility on shaft whirl stability, ignoring completely the flexibility of the two end bearings. The third discusses only the elastic shaft without an attached appendage on eight-coefficient bearings.

The results of a literature review thus prompted this study on parametric stability analysis of rotor-bearing systems via Liapunov's direct method. A general method of analysis is presented in this work to inves-

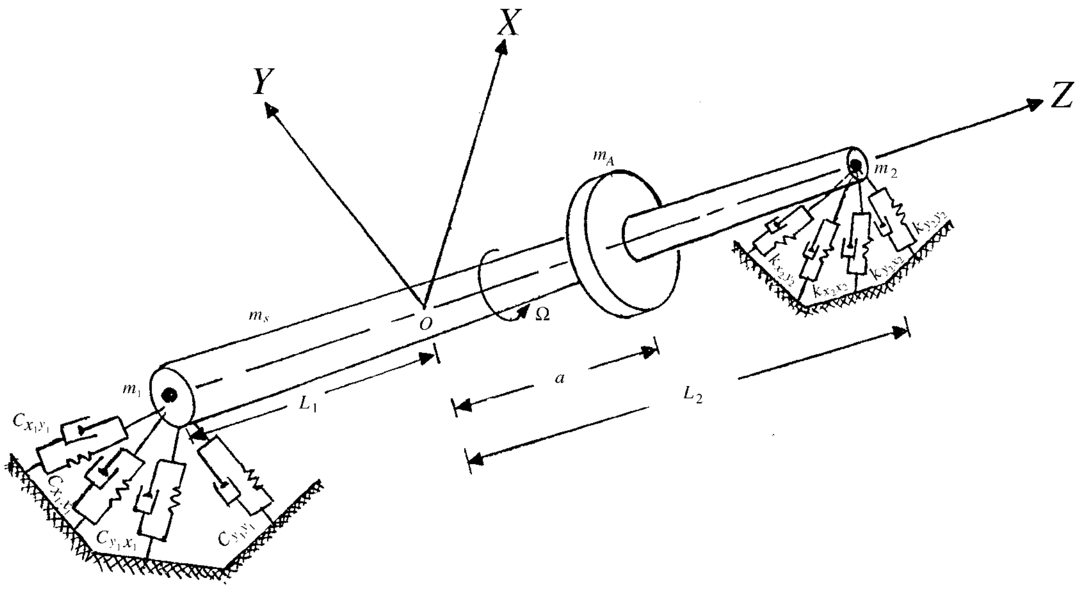

Fig. 1. Rotor bearing model.

tigate how different system parameters can affect the stability of its whirling motion. This is done through a model of an axially symmetric rigid appendage attached at an arbitrary location along a nonuniform rigid rotating shaft mounted on two dissimilar eight-component end bearings. A set of sufficient conditions of asymptotic stability is obtained as a function of various system parameters. Stability boundaries are presented in graphical forms in terms of system nondimensionalized parameters.

II. Problem Formulation

The model consists of an axially symmetric rigid appendage of mass m_A rigidly attached at an arbitrary location along a shaft of mass m_S and length 2ℓ, which is supported at its ends by two dissimilar eight-coefficient bearings as shown schematically in Fig. 1. The static deflections are considered negligible compared to the dynamics effect, and aerodynamic forces are not included.

Consider that in the dynamic equilibrium configuration of the system, the shaft is along the z direction of a rotating x,y,z coordinate system located at the shaft center of mass and described by unit vectors $\hat{i}, \hat{j}, \hat{k}$. Denoting the displacement of the shaft center of mass by

$$\vec{r}_s = x(t)\hat{i} + y(t)\hat{j}\,, \tag{1}$$

we can describe the position vector $\vec{r}_1$ of the center of the left bearing and $\vec{r}_2$ of the right bearing center by

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} - \ell_1\hat{k} \tag{2}$$

and

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + \ell_2\hat{k}\,, \tag{3}$$

where for small angular and translational displacements, we have

$$\theta = \frac{x - x_1}{\ell_1} = \frac{x_2 - x}{\ell_2} \tag{4}$$

and

$$\phi = \frac{y - y_1}{\ell_1} = \frac{y_2 - y}{\ell_2}\,, \tag{5}$$

in which ℓ_1 and ℓ_2 are the distances from the shaft center of mass to the left and right bearings respectively. Also, the position vector of the appendage center of mass at a distance, a, from the shaft mass center is given by

$$\vec{r}_A = (x + a\theta)\hat{i} + (y + a\phi)\hat{j} + a\hat{k}\,. \tag{6}$$

Using the infinitesimal rotation concept (where second order terms of ε_i are neglected), the resultant angular velocity of the appendage-shaft system $\vec{\varpi} = \vec{\Omega} + \vec{\dot{\theta}} + \vec{\dot{\phi}}$ can be written as

$$\vec{\varpi} = (\dot{\phi} - \theta\Omega)\hat{i} + (\dot{\theta} - \phi\Omega)\hat{j} + (\Omega + \theta\dot{\phi})\hat{k}\,, \tag{7}$$

where Ω is the spinning speed of the system.

Differentiating Eqs. (1), (2), (3) and (6) with respect to time, the velocity vectors $\dot{\vec{r}}_s$, $\dot{\vec{r}}_1$, $\dot{\vec{r}}_2$ and $\dot{\vec{r}}_A$ are given by

$$\dot{\vec{r}}_s = [\dot{x} - y(\Omega + \theta\dot{\phi})]\hat{i} + [\dot{y} + x(\Omega + \theta\dot{\phi})]\hat{j} + [y(\dot{\phi} - \theta\Omega) - x(\dot{\theta} - \phi\Omega)]\hat{k}\,, \quad \textbf{(8)}$$

$$\dot{\vec{r}}_1 = [\dot{x} - \ell_1\dot{\theta} + (y - \ell_1\phi)(\Omega + \theta\dot{\phi}) - \ell_1(\dot{\theta} - \phi\Omega)]\hat{i} + [\dot{y} - \ell_1\dot{\phi} + (x - \ell_1\theta)(\Omega + \theta\dot{\phi}) - \ell_1(\dot{\phi} - \theta\Omega)]\hat{j} + [(y - \ell_1\phi)(\dot{\phi} - \theta\Omega) - (x - \ell_1\theta)(\dot{\theta} - \phi\Omega)]\hat{k}\,, \quad \textbf{(9)}$$

$$\dot{\vec{r}}_2 = [\dot{x} + \ell_2\dot{\theta} - (y + \ell_2\phi)(\Omega + \theta\dot{\phi}) + \ell_2(\dot{\theta} - \phi\Omega)]\hat{i} + [\dot{y} + \ell_2\dot{\phi} + (x + \ell_2\theta)(\Omega + \theta\dot{\phi}) - \ell_2(\dot{\phi} - \theta\Omega)]\hat{j} + [(y + \ell_2\phi)(\dot{\phi} - \theta\Omega) - (x + \ell_2\theta)(\dot{\theta} - \phi\Omega)]\hat{k}\,, \quad \textbf{(10)}$$

and

$$\dot{\vec{r}}_A = [\dot{x} + a\dot{\theta} - (y + a\phi)(\Omega + \theta\dot{\phi}) + a(\dot{\theta} - \phi\Omega)]\hat{i} + [\dot{y} + a\dot{\phi} + (x + a\theta)(\Omega + \theta\dot{\phi}) - a(\dot{\phi} - \theta\Omega)]\hat{j} + [(y + a\phi)(\dot{\phi} - \theta\Omega) - (x + a\theta) + (\dot{\theta} - \phi\Omega)]\hat{k}\,. \quad \textbf{(11)}$$

The kinetic energy of the system is

$$T = \frac{1}{2}[m_s\dot{\vec{r}}_s \bullet \dot{\vec{r}}_s + m_A\dot{\vec{r}}_A \bullet \dot{\vec{r}}_A + m_1\dot{\vec{r}}_1 \bullet \dot{\vec{r}}_1 + m_2\dot{\vec{r}}_2 \bullet \dot{\vec{r}}_2 + I_d(\omega_x^2 + \omega_y^2) + I_p\omega_z^2]\,, \quad \textbf{(12)}$$

where m_1 and m_2 are the masses of the left and right bearings, respectively, whereas I_d and I_p are the diametral and polar mass moments of inertia of the appendage–shaft system.

The strain energy expression for the bearing system under consideration is

$$V = \frac{1}{2}\sum_i\sum_j k_{ij}q_iq_j = \frac{1}{2}(k_{x_1x_1}x_1^2 + k_{x_1y_1}x_1y_1 + k_{y_1y_1}y_1^2 + k_{y_1x_1}y_1x_1 + k_{x_2x_2}x_2^2 + k_{x_2y_2}x_2y_2 + k_{y_2y_2}y_2^2 + k_{y_2x_2}y_2x_2)\,, \quad \textbf{(13)}$$

in which the subscript 1 refers to the left bearing and 2 to the right bearing.

The energy dissipation function of the system is

$$D = \frac{1}{2}\sum_i\sum_j c_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}(c_{x_1x_1}\dot{x}_1^2 + c_{x_1y_1}\dot{x}_1\dot{y}_1 + c_{y_1y_1}\dot{y}_1^2 + c_{y_1x_1}\dot{y}_1\dot{x}_1 + c_{x_2x_2}\dot{x}_2^2 + c_{x_2y_2}\dot{x}_2\dot{y}_2 + c_{y_2y_2}\dot{y}_2^2 + c_{y_2x_2}\dot{y}_2\dot{x}_2)\,. \quad \textbf{(14)}$$

The state of motion of the dynamical system under consideration is completely defined by the four state variables x, y, θ and ϕ.

III. Stability Analysis: General Case

Lagranges equations,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i\,,\ i=1,\ 2,\ 3,\ 4, \quad \textbf{(15)}$$

can be used to derive the system equations of motion where the generalized non-conservative force Q_i of the system is a damping force of Rayleigh's type, defined by

$$Q_i = \frac{-\partial D}{\partial\dot{q}_i}\,, \quad \textbf{(16)}$$

and the Lagrangian L is

$$L=T-v.$$

However, the resulting equations of motion are four coupled nonlinear second order differential equations that are impossible to solve analytically. Therefore, stability analysis is adopted in this investigation to determine the behavior of the nonlinear system of equations in the neighborhood of the equilibrium configuration, identified by $q_i=\dot{q}_i=0$, based on Liapunov's direct method. If damping is neglected, the Hamiltonian H is a constant called the Jacobi integral, which is well known to be a suitable Liapunov function. In the presence of damping, the total time derivative of H is

$$\dot{H} = \sum_{i=1}^{n}\frac{\partial H}{\partial q_i}\dot{q}_i + \frac{\partial H}{\partial p_i}\dot{p}_i\,, \quad \textbf{(17)}$$

where p_i is the generalized momentum of the system.

In view of the following Hamiltanian equations of the considered system:

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \text{ and } \dot{p}_i = -\frac{\partial H}{\partial q_i} + Q_i\,, \quad \textbf{(18)}$$

Eq. (17) becomes

$$\dot{H} = \sum_{i=1}^{n} Q_i\dot{q}_i\,. \quad \textbf{(19)}$$

Equation (19) indicates that the sign definiteness of H depends on Q_i. Since Q_i comprises damping forces representing either complete damping or pervasive damping, it does not alter the nature of equilibrium in a meaningful way. That is to say, a stable system becomes asymptotically stable whereas an unstable one remains unstable. Therefore, for such a system, the Hamiltonion still serves as a suitable Liapunov function.

The system Hamiltonion function can be written as

$$H=T_2-T_0+V, \tag{20}$$

in which the subscriptes of T denote the degree of homogenity in the generalized velocity variables $\dot{q}_i$.

Since T_2 is by definition positive definite in $\dot{q}_i$, and since V and T_0 depend only on q_i, the positive definiteness of the dynamic potential U given by

$$U=-T_0+V \tag{21}$$

ensures the positive definiteness of H. Thus, the following function U serves as a suitable Liapunov testing function for the dynamical system under consideration:

$$\begin{aligned} U = \tfrac{1}{2}\{ & k_{x_1x_1}(x-\ell_1\theta)^2 + (k_{x_1y_1}+k_{y_1x_1})(x-\ell_1\theta)(y-\ell_1\theta) \\ & + k_{y_1y_1}(y-\ell_1\phi)^2 + k_{x_2x_2}(x+\ell_2\phi)^2 + (k_{x_2y_2}+k_{y_2x_2}) \\ & \bullet (x+\ell_2\theta)(y+\ell_2\theta) + k_{y_2y_2}(y+\ell_2\phi)^2 - I_d\Omega^2(\theta^2+\phi^2) \\ & - I_p\Omega^2 - m_1\Omega^2[\ell_1^2(\theta^2+\phi^2) + (x-\ell_1\theta)^2(1+\phi^2) \\ & - (y-\ell_1\phi)^2(1+\theta^2) - 2\ell_1\theta(x-\ell_1\theta) - 2\ell_1\phi(y-\ell_1\phi) \\ & - 2\theta\phi(x-\ell_1\theta)(y-\ell_1\phi)] - m_s\Omega^2(x^2+y^2+(x\phi-y\theta)^2) \\ & - m_A\Omega^2[(1+\phi^2)(x+a\theta)^2 + (1+\theta^2)(y+a\phi)^2 \\ & + a^2(\theta^2+\phi^2) - 2a\theta(x+a\theta) - 2a\phi(y+a\phi) \\ & - 2\theta\phi(x+a\theta)(y+a\phi)] - m_2\Omega^2[\ell_2^2(\theta^2+\phi^2) \\ & + (1+\phi^2)(x+\ell_2\theta)^2 + (1+\theta^2)(y+\ell_2\phi)^2 \\ & - 2\theta\phi(x+\ell_2\theta)(y+\ell_2\phi) + 2\ell_2\theta(x+\ell_2\theta) \\ & + 2\ell_2\phi(y+\ell_2\phi)]\} . \end{aligned} \tag{22}$$

In view of Eqs. (14) and (16) and using Euler's theorem of homogenous functions, Eq. (19) becomes

$$\dot{H}=-2D. \tag{23}$$

Thus, a positive definite D leads to a negative definite $\dot{H}$. Hence, according to the Liapunov stability theory, if for such a system H is positive definite (which can be guaranteed by the positive definiteness of U), then the system is asymptotically stable; and if H can assume negative values in the neighborhood of the equilibrium point, then the system is unstable. Thus, the problem simplifies to testing for (1) the positive definiteness of U to achieve stability in the undamped case and (2) the positive definiteness of both U and D in order to achieve asymptotic stability of the damped system. Testing for the positive definiteness of U and D can be accomplished by applying Sylvester's theorem to the Hessian matrices of U and D (evaluated at the equilibrium point) given by

$$U_{ij} = \left.\frac{\partial^2 U}{\partial q_i \partial q_j}\right|_E$$

and **(24)**

$$D_{ij} = \left.\frac{\partial^2 D}{\partial \dot{q}_i \partial \dot{q}_j}\right|_E .$$

IV. Similar Bearings and a Uniform Shaft

To demonstrate the general method of analysis, the bearings are taken to be similar (i.e., $k_{i_1j_1}= k_{i_2j_2}=\frac{1}{2}k_{ij}$, $c_{i_1j_1}=c_{i_2j_2}=\frac{1}{2}c_{ij}$, and $m_1=m_2=\frac{1}{2}m'$), and the shaft is made uniform with its mass center coincident with the appendage mass center (i.e., $\ell_1=\ell_2=\ell$ and a=0) in order to simplify the complexity of the stability calculations and the resulting conditions. Dividing U by $m\Omega^2$ (where $m=m'+m_A+m_S$), evaluating the second partial derivatives at the equilibrium point and making appropriate algebraic manipulations, the Hessian matrix U_{ij} can be written as

$$U_{ij}=\begin{bmatrix} \left[\frac{\omega_{xx}^2}{\Omega^2}-1\right] & \alpha & 0 & 0 \\ \alpha & \left[\frac{\omega_{yy}^2}{\Omega^2}-1\right] & 0 & 0 \\ 0 & 0 & \ell^2\left[\frac{\omega_{xx}^2}{\Omega^2}-C\right] & \ell^2\alpha \\ 0 & 0 & \ell^2\alpha & \ell^2\left[\frac{\omega_{yy}^2}{\Omega^2}-C\right] \end{bmatrix}, \tag{25}$$

in which the nondimensional parameters C and α are defined by

$$C = 4\frac{m'}{m} + \frac{I_d}{m\ell^2} \tag{26}$$

and

$$\alpha = (\omega_{xy}^2 + \omega_{yx}^2)/2\Omega^2 \tag{27}$$

where $\omega_{ij}^2 = k_{ij}/m$.

Applying Sylvester's Theorem to test the positive definiteness of U_{ij}, its successive principal minor determinants must be positive definite, thus yielding the following sufficient conditions for system whirl stability:

$$\frac{\omega_{xx}^2}{\Omega^2} > 1\ , \tag{28}$$

$$\left[\frac{\omega_{xx}^2}{\Omega^2} - 1\right]\left[\frac{\omega_{yy}^2}{\Omega^2} - 1\right] > \alpha^2\ , \tag{29}$$

$$\frac{\omega_{xx}^2}{\Omega^2} > C \tag{30}$$

and

$$\left[\frac{\omega_{xx}^2}{\Omega^2} - C\right]\left[\frac{\omega_{yy}^2}{\Omega^2} - C\right] > \alpha^2\ . \tag{31}$$

V. Discussion of Results

The following points are made based on an examination of the foregoing conditions;

(1) Conditions **(28)-(31)** represent sufficient conditions for whirl stability of an axially symmetric rigid appendage on the midspan of a uniform spinning shaft mounted on elastic bearings possessing mass and both anisotropic and cross-coupling stiffness coefficients.

(2) For $C \geq 1$, it is seen that once conditions **(30)** and **(31)** are satisfied, conditions **(28)** and **(29)** are automatically satisfied. In other words, the system stability criterion can be performed based on conditions **(30)** and **(31)** only, leaving conditions **(28)** and **(29)** as trivial stability conditions in this case. On the other hand, if $C \leq 1$, it is clear that satisfaction of conditions **(28)** and **(29)** implies satisfaction of conditions **(30)** and **(31)**. Therefore, in the case of $C \leq 1$, conditions **(28)** and **(29)** constitute the basis of the system stability criterion whereas conditions **(30)** and **(31)** are considered as trivial conditions.

(3) The stability criteria in conditions **(28)-(31)** functionally show the effects of the system's nondimensional parameters $\frac{\omega_{xx}^2}{\Omega^2}$, $\frac{\omega_{yy}^2}{\Omega^2}$, α anc C on the whirl stability of the dynamical system under investigation. The effects of these factors on the stability boundaries of the system based on conditions **(28)-(31)** and the argument in the foregoing point are presented graphically in Figs. 2-5.

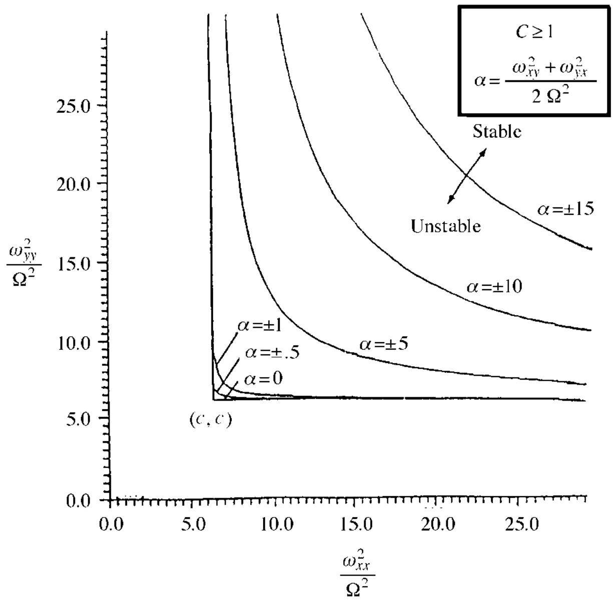

Fig. 2. Effect of the nondimentional cross-coupling parameter on stability regions for $C \geq 1$.

(4) Figures 2 and 3 clearly demonstrate the influence of the bearing cross-coupling nondimensionalized parameter $\alpha = (\frac{\omega_{xy}^2 + \omega_{yx}^2}{2\Omega^2})$ as a serious source of instability where it is easy to see that the curves with higher values of α contain smaller stability regions.

(5) To demonstrate the effect of the principal stiffness coefficients on the system stability regions, let us take $(\frac{\omega_{xx}^2}{\Omega^2} - 1)$ in conditions **(28)-(31)** as a parameterdenoted by λ. The system stability boundaries, drawn in terms of λ, are then represented by the family of parabolas shown in Figs. 4 and 5. It is clear from Figs. 4 and 5 that the greater the value of λ, the larger the region of stability. The maximum stability region in the figures is found when λ tends to infinity (rigid bearings), where the stability region tends to be the entire area above the common tangential horizontal line $\frac{\omega_{yy}^2}{\Omega^2} = 1$ or $\frac{\omega_{yy}^2}{\Omega^2} = C$. This supports the intuitive expectation that the increase

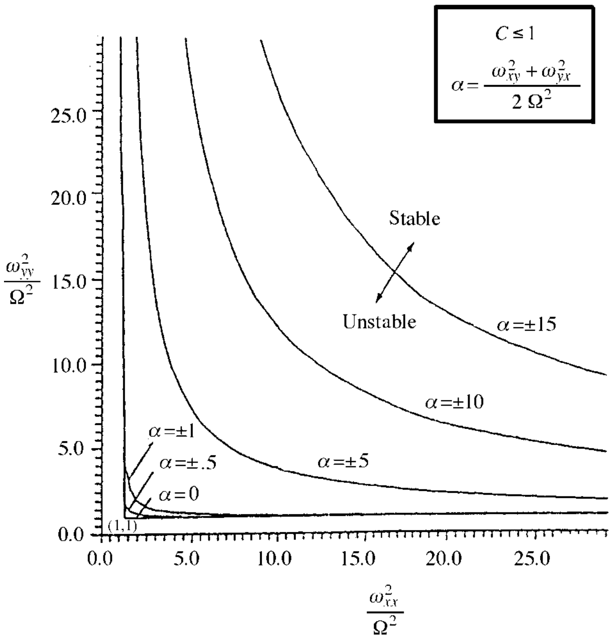

Fig. 3. Effect of the nondimentional cross-coupling parameter on stability regions for $C \leq 1$.

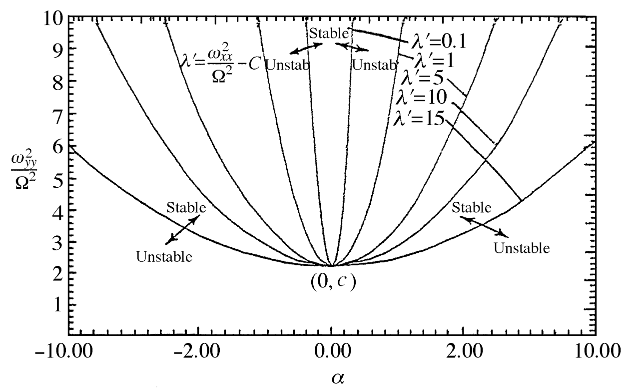

Fig. 4. Effect of the nondimensional principal stiffness parameter λ' on stability regions for $C \geq 1$.

of bearing rigidity enhances the dynamic stability of the system.

(6) Based on the argument in point (2) and the results illustrated in Figs. 2–5, it is seen that the system stability boundaries are not affected by the nondimensional parameter C in the case where the value of this parameter is less than or equal to unity. On the other hand, it is shown that when $C>1$, greater values of C lead to smaller stability regions. Thus, the role that can be played by the bearing mass ratio and the appendage mass and dimensions in affecting system stability can be demonstrated if we recall that $C=\frac{4m'}{m}+\frac{I_d}{m\ell^2}$.

It is then obvious that the value of the bearing mass ratio $\frac{m'}{m}$ should be less than 0.25 such that $C>1$ in order to guarantee that the bearing mass has no influence on system whirl stability. However, if the bearing mass ratio $\frac{m'}{m}$ is greater than or equal to 0.25, then it is certain that the bearing mass will have a bad effect on system stability. It is also clear that if the mass and the dimensions of the appendage are such that the value of the ratio $\frac{I_d}{m\ell^2} \leq (1-\frac{4m'}{m})$, then they will not affect system whirl stability; otherwise, they will have had influence on system stability.

VI. Effect of Damping

A necessary condition to achieve asymptotic stability of the damped system in the sense of Liapunov is the positive definiteness of D since it leads to the negative definiteness of $\dot{H}$. Therefore, sufficient conditions for asymptotic stability require satisfaction of the positive definiteness of D_{ij} in addition to the positive definiteness of U_{ij}. Therefore, letting $C_{i_1j_1}=C_{i_2j_2}=\frac{1}{2}C_{ij}$ in Eq. (14) for similar bearings and evaluating the second partial derivatives of D at the equilibrium point $q_i=\dot{q}_i=0$, the Hession matrix D_{ij} can be put on the form

$$D_{ij} = \begin{bmatrix} C_{xx} & \gamma & 0 & 0 \\ \gamma & C_{yy} & 0 & 0 \\ 0 & 0 & \ell^2 C_{xx} & \ell^2\gamma \\ 0 & 0 & \ell^2\gamma & \ell^2 C_{yy} \end{bmatrix}, \tag{32}$$

in which the bearing cross-coupling damping parameter γ is defined by

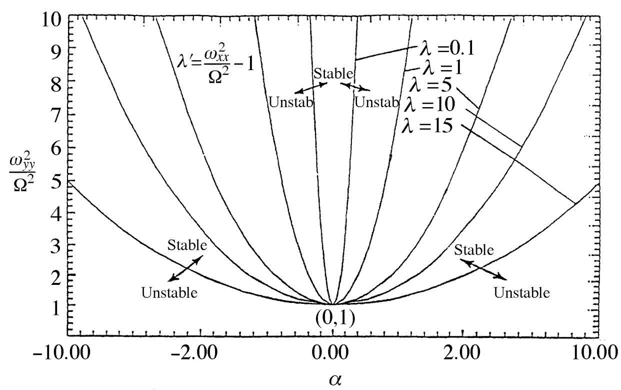

Fig. 5. Effect of the nondimensional principal stiffness parameter λ on stability regions for $C \leq 1$.

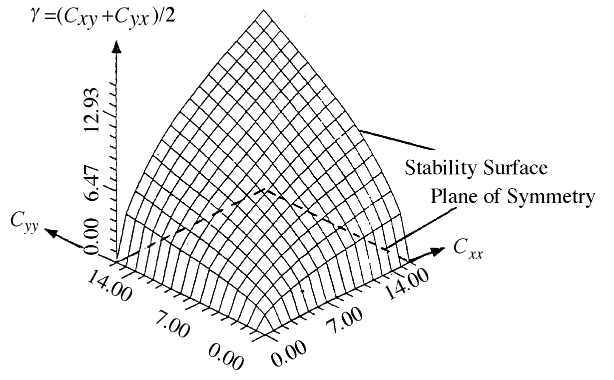

Fig. 6. Stability surface of the bearing damping coefficients space.

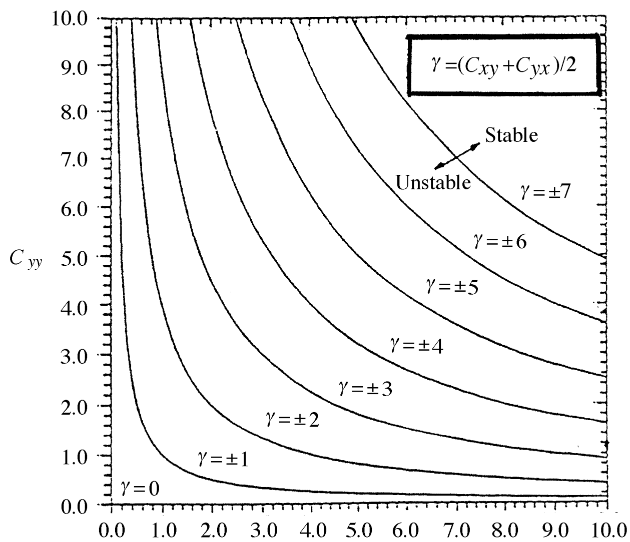

Fig. 7. Effect of γ on stability regions.

$$\gamma=\frac{1}{2}(C_{xy}+C_{yx})\,. \tag{33}$$

Application of Sylvester's theorem to test the positive definiteness of D_{ij} yields the following non-trivial necessary conditions for an asymptotically stable damped system;

$$C_{xx}>0$$

and

$$C_{xx}C_{yy}>\gamma^2. \tag{34}$$

If a three dimensional space for the damping coefficients is constructed using C_{xx}, C_{yy} and γ as a basis for this space, then condition **(34)** can be geometrically visualized. Accordingly the subspace of the stable bearing damping coefficients based on condition **(34)** is the spatial region inside the parabolic hyperbolid shown in Fig. 6. This indicates that all possible values of bearing damping coefficients that result in asymptotically stable motion are located inside this stability surface. The cross sections of the stability surface in Fig. 6 formed by planes perpendicular to either C_{xx} or C_{yy} is a family of parabolas. Thus, from Fig. 6, it is easy to see that the stability regions grow with the increase of the principal damping coefficients C_{xx} or C_{yy}. It can also be shown that if the stability surface is cut by planes perpendicular to the γ-axis, then the cross-sections are rectangular hyperbolas representing the stability boundaries in terms of γ as shown in Fig. 7. The area inside each hyperbola decreases with the increase of γ, which indicates that the stability regions decrease with the increase of γ. This clearly illustrates the significance of the bearing cross-coupling damping coefficients as a major source of whirl instability of rotor-bearing systems. This result agrees with the results reached before by the author (El-Marhomy, 1995, 1997) and others such as Gunter (1966) and Rao (1983, 1984). The significance of the C_{ij} coefficient in harming rotor stability may be physically attributed to the fact that it produces a force in the $\hat{i}$ direction due to a time rate of change of a displacement from the equilibrium configuration in the perpendicular direction $\hat{j}$ leading to self-excited rotor instability.

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轉子—軸承系統之參數穩定分析

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摘　要

本文介紹一種運用Liapunov直接法的通用分析方法，研究一個代表轉子-軸承系統在平衡點鄰域之行為的非線性微分方程系統。文中建立一個兩端固定於不同的八部件軸承上的非均勻轉軸的數學模式，此模式包含可附加在轉軸上任意位置的軸對稱剛性附加物，以此數學模式發展出針對不同系統參數的穩定性準則。經由本文所得到的方程式利用無因次參數作圖以表示穩定邊界，是提供工程設計資料的典型案例。本文的結果顯示了(1)軸承質量、(2)附加質量及其尺寸、(3)軸承主勁度與阻尼係數、以及(4)軸承交互勁度與阻尼係數等參數在影響系統旋轉穩定性中所扮演的角色。