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Economic Analysis of Inventory System for Deteriorating Items with Shortages and a Finite Planning Horizon

KUN-JEN CHUNG AND JOSEPH LIU

Department of Industrial Management
National Taiwan Institute of Technology
Taipei, Taiwan, R.O.C.

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ABSTRACT

We consider here the inventory replenishment policy over a fixed planning period for a deteriorating item having a constant demand with shortages. This paper also presents a line search technique adopting the discounted cash flow approach to decide on the optimal interval which has positive inventories. One numerical example illustrates how the procedure works.

Key Words: EOQ models, deterioration, shortages, discounted cash flow

I. Introduction

Usually, analysis of inventory systems is carried out without considering the effects of deterioration. Fortunately, for most items, the rate of deterioration is so low that there is little need for consideration of deterioration in determining economic lot sizes. However, there are items such as highly volatile substances, radioactive materials etc. for which the rate of deterioration is very large. In addition, in a few fields, such as perishable foods and the production of certain chemicals and electronic components, significant deterioration may occur during normal storage periods. Therefore, loss from deterioration should not be ignored.

In general, inventory models in which items deteriorate while in storage can be broadly divided into two catagories:

(1) Inventory models with items having a fixed life time: These items are called perishable items.

(2) Inventory models with items having a stochastic life time: These items are called deteriorating items.

This paper only is concerned with deteriorating items. Nahmias (1982) provided a reference list of 77 periodicals and books dealing with ordering policies for perishable inventories. Raafat (1991) presented a complete and up-to-date survey of the published inventory literature for deteriorating inventory models. The above literature reveals that perishable and deteriorating models have received particular attention, and that a considerable amount of work has been done.

The first attempt to describe optimal policies for deteriorating items was made by Ghare and Schrader (1963), who derived a revised form of the economic order quantity (EOQ) model assuming exponential decay. This model was extended to consider Weibull distribution deterioration by Covert and Philip (1973), and was further extended to the case of general distribution deterioration by Shah (1977). Dave (1979) considered a deterministic order level inventory model continuous in units and discrete in time. Dave and Patel (1981) developed an inventory model for deteriorating items with time-varying demand and presented analysis for a linear increasing trend only. Dave and Patel's model was extended by Sachan (1984) to cover the backlogging option. Bahari-Kashani (1989) suggested a heuristic to deal with it. Chung and Ting (1994) revealed a heuristic which adjusted and improved the approximate procedure in the Bahari-Kashani model. Recently, Ting and Chung (1994) studied the inventory replenishment model for deteriorating items with a linear trend in demand considering shortages. On the other hand, Hollier and Mak (1983) considered a demand rate which was decreasing negative exponentially and obtained optimal replenishment policies under both constant and variable replenishment intervals. Aggarwal and Bahari-Kashani (1991) discussed a production-inventory system for a line of products in which the items were deteriorating at a constant rate, the demand rate decreased negative exponentially, and no shortages were allowed. Goswami and Chaudhuri (1991) studied the inventory replenishment policy over a fixed planning period for a deteriorating item having a deterministic demand pattern with a linear trend and shortages.

In all of the above mentioned models, the time value of money was disregarded. Generally speaking, although time value corrections for time intervals of less than a year are considered to be relatively small, we can not conclude that time value corrections are not needed since the contribution of time value to inventory management depend upon the kind of model, the length of the inventory cycle and the type of interest rate. Therefore, it is important to investigate how time value influences various inventory policies. Trippi and Lewin (1974) adopted a discounted cash flow (DCF) approach to obtain the present value of average inventory costs over an infinite horizon. Dohi *et al.* (1992) discussed inventory systems without and with backlogging allowed for an infinite time span, taking into account time value from a viewpoint different from that of Trippi and Lewin (1974). Moon and Yun (1993) employed the DCF approach to fully recognize the time value of money to develop a finite planning horizon EOQ model where the planning horizon is a random variable. Hariga (1994) studied the effects of inflation and the time value of money on the replenishment policies of items with time continuous non-stationary demand over a finite planning horizon. Although all the models mentioned in this paragraph consider the time value of money, they do not consider deterioration. The above literature also reveals that both the finite horizon and the infinite planning horizon are simultaneously adopted in the inventory system. Gurnani (1983) applied the DCF approach to the finite-planning horizon EOQ model, in which the planning horizon is a given constant, and indicated that an infinite planning horizon rarely occurs in practice because costs are likely to vary disproportionately and because of changes in product specifications and design when a product is discontinued or replaced by another product due to rapid technological development. Chung and Kim (1989) also suggested that the assumption of an infinite planning horizon is not realistic and called for a new model which relaxes the assumption of an infinite planning horizon. The objective of this study was to develop inventory models having constant demand with shortages over a finite planning horizon, from which optimal policies could be obtained while giving due consideration to deterioration effects and the time value of money. We apply the DCF approach to determine the optimal number of replenishments to be made and the corresponding cycle length, consisting of positive and negative inventories periods. Furthermore, this paper presents a procedure which uses a line search method to approximate the optimal cycle interval which has positive inventories. Finally, a numerical example is also given to illustrate the procedure.

II. The Mathematical Model and Analysis

We need the following notations and assumptions to develop the model.

(1) A single item is considered over a prescribed period of H units of time.

(2) The demand rate, D units per year, is known and constant.

(3) The rate of replenishments is infinite; the replenishment interval is constant; the lead time is zero. At the beginning of every inventory cycle, a constant lot-size Q is ordered to meet demands during replenishment interval T except for the final cycle because the backlogged demands of the first cycle are met by the second replenishment, and the last cycle do not allow shortages.

(4) Shortages are allowed except for in the final cycle. Shortages are also assumed to be completely backlogged and filled as soon as fresh stock arrives.

(5) The constant rate of deterioration, θ, is only applied to on-hand inventory, and there is no replacement or repair of deteriorated units during the period H.

(6) The relevant costs are the inventory carrying cost h per unit per year applied to good units only, the shortage cost π per unit per year, the deterioration cost c per unit, and the replenishment cost S per order, which are all known and constant during the period H.

(7) The inventory carrying cost, shortage cost and deterioration cost are assumed to be proportional to the inventory level and are incurred instantaneously.

(8) A DCF approach is adopted to consider the time value of money. The discount rate α is compounded continuously.

(9) The order quantity, inventory level, deterioration and demand are treated as continuous variables, and the number of replenishments is treated as a discrete variable.

The total time horizon H is divided into m equal parts of length T, so that $T=H/m$. The reorder times over the time horizon H will be jT (j=0, 1, 2, ..., $m-1$). Initial and final inventories are both zero. The inventory system is described graphically in Fig. 1. Figure 1 reveals that each inventory cycle except the

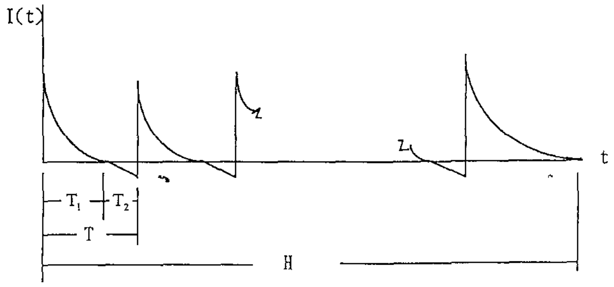

Fig. 1. State diagram of the inventory system for one cycle.

last cycle can be divided into two subperiods of length T_1 and T_2, respectively. We assume that the period for which there are positive inventories in each interval is $[jT, jT+T_1]$ and the period for which there are negative inventories in each interval is $[jT+T_1, (j+1)T]$ where j=0, 1, 2, ..., $m-2$. In each interval $[jT, (j+1)T]$, shortages occur at time $jT+T_1$, where j=0, 1, 2, ..., $m-2$. The last replenishment occurs at time $(m-1)T$, and shortages are not allowed in the last period $[(m-1)T, H]$.

The problem is to obtain optimal values of m and T_1 which will minimize the cost over the finite planning horizon $[0, H]$. The inventory level of the system at time t, $I(t)$, over period $[0, T]$ can be described by the following equations:

$$\frac{dI(t)}{dt} + \theta I(t) = -D \qquad 0 \le t \le T_1, \tag{1}$$

$$\frac{dI(t)}{dt} = -D \qquad T_1 \le t \le T_1+T_2=T. \tag{2}$$

The solution of the above differential equations using boundary condition $I(T_1)=0$ is

$$I(t) = \frac{D}{\theta}\left[\frac{e^{\theta T_1} - e^{\theta t}}{e^{\theta t}}\right], \qquad 0 \le t \le T_1, \tag{3}$$

$$I(t) = -D(t-T_1), \qquad T_1 \le t \le T_1+T_2=T. \tag{4}$$

The present value of the inventory carrying cost: The inventory carrying cost at time t is $hI(t)$. The present value of the inventory carrying cost is obtained by discounting $hI(t)$ at a rate of α, i.e. $hI(t)e^{-\alpha t}$. Hence, the present value of the inventory carrying cost for the first cycle is

$$h\int_0^{T_1} \frac{D}{\theta}\left[\frac{e^{\theta T_1} - e^{\theta t}}{e^{\theta t}}\right] e^{-\alpha t} dt. \tag{5}$$

Equation (5) can be simplified as

$$\frac{hD}{\theta}\left\{\frac{e^{\theta T_1}}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)T_1}\right] + \frac{1}{\alpha}\left(e^{-\alpha T_1} - 1\right)\right\}. \tag{6}$$

The present value of the deterioration cost: The number of deteriorated units is equal to $\theta I(t)$. Thus, the present value of the deterioration cost for the first cycle is

$$c\int_0^{T_1} D\left[\frac{e^{\theta T_1} - e^{\theta t}}{e^{\theta t}}\right] e^{-\alpha t} dt$$

$$= cD\left\{\frac{e^{\theta T_1}}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)T_1}\right] + \frac{1}{\alpha}\left(e^{-\alpha T_1} - 1\right)\right\}. \tag{7}$$

The present value of the shortage cost: According to the above arguments, the present value of the shortage cost for the first cycle can be shown as

$$\pi\int_0^{T_2} Dt e^{-\alpha(T_1+t)} dt$$

$$= -\frac{\pi D}{\alpha} e^{-\alpha T_1}\left[T_2 e^{-\alpha T_2} + \frac{1}{\alpha}\left(e^{-\alpha T_2} - 1\right)\right]. \tag{8}$$

Under the assumption that there are m replenishments over $[0, H]$,

the total cost for the first cycle
= replenishment cost + inventory carrying cost
+ deterioration cost + shortage cost

$$= S + \left(\frac{hD}{\theta} + cD\right)\left\{\frac{e^{\theta T_1}}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)T_1}\right] + \frac{1}{\alpha}\left(e^{-\alpha T_1} - 1\right)\right\}$$

$$- \frac{\pi D}{\alpha} e^{-\alpha T_1}\left[(T - T_1)e^{-\alpha(T-T_1)} + \frac{1}{\alpha}\left(e^{-\alpha(T-T_1)} - 1\right)\right]. \tag{9}$$

Equation (9) can be simplified as

the total cost for the first cycle

$$= \frac{hD + cD\theta}{\theta(\alpha+\theta)} e^{\theta T_1} + \frac{D\alpha(h + c\theta) + \pi D(\alpha+\theta)}{\alpha^2(\alpha+\theta)} e^{-\alpha T_1}$$

$$+ \frac{\pi D}{\alpha} T_1 e^{-\alpha T} - \frac{\pi D}{\alpha} e^{-\alpha T}\left(T + \frac{1}{\alpha}\right) + \left(S - \frac{hD + cD\theta}{\alpha\theta}\right). \tag{10}$$

Therefore, the present value of the total cost during the first $(m-1)$ inventory cycles can be written as follows:

$$\sum_{i=0}^{m-2} e^{-i\alpha T}\left\{\frac{hD + cD\theta}{\theta(\alpha+\theta)} e^{\theta T_1} + \frac{D\alpha(h + c\theta) + \pi D(\alpha+\theta)}{\alpha^2(\alpha+\theta)} e^{-\alpha T_1}\right.$$

$$\left. + \frac{\pi D}{\alpha} T_1 e^{-\alpha T} - \frac{\pi D}{\alpha} e^{-\alpha T}\left(T + \frac{1}{\alpha}\right) + \left(S - \frac{hD + cD\theta}{\alpha\theta}\right)\right\}$$

$$= \frac{1 - e^{-\alpha(m-1)T}}{1 - e^{-\alpha T}}\left\{\frac{hD + cD\theta}{\theta(\alpha+\theta)} e^{\theta T_1}\right.$$

$$+\frac{D\alpha(h+c\theta)+\pi D(\alpha+\theta)}{\alpha^2(\alpha+\theta)}e^{-\alpha T_1}$$

$$+\frac{\pi D}{\alpha}T_1 e^{-\alpha T}-\frac{\pi D}{\alpha}e^{-\alpha T}(T+\frac{1}{\alpha})+(S-\frac{hD+cD\theta}{\alpha\theta})\}. \quad \mathbf{(11)}$$

Because shortages are not allowed in the last cycle, based on the previous discussion, the total cost during the last cycle is

$$S+(\frac{hD}{\theta}+cD)\{\frac{e^{\theta T}}{\alpha+\theta}[1-e^{-(\alpha+\theta)T}]+\frac{1}{\alpha}(e^{-\alpha T}-1)\}. \quad \mathbf{(12)}$$

Hence, the present value of the total cost during the last cycle is

$$e^{-\alpha(m-1)T}\{S+(\frac{hD}{\theta}+cD)[\frac{e^{\theta T}}{\alpha+\theta}(1-e^{-(\alpha+\theta)T})$$

$$+\frac{1}{\alpha}(e^{-\alpha T}-1)]\}. \quad \mathbf{(13)}$$

Consequently, the present value of the total cost over the entire planning horizon $TC(m, T_1)$ is

$$TC(m,T_1)$$

$$=\frac{1-e^{-\alpha(m-1)T}}{1-e^{-\alpha T}}\{\frac{hD+cD\theta}{\theta(\alpha+\theta)}e^{\theta T_1}$$

$$+\frac{D\alpha(h+c\theta)+\pi D(\alpha+\theta)}{\alpha^2(\alpha+\theta)}e^{-\alpha T_1}$$

$$+\frac{\pi D}{\alpha}T_1 e^{-\alpha T}-\frac{\pi D}{\alpha}e^{-\alpha T}(T+\frac{1}{\alpha})+(S-\frac{hD+cD\theta}{\alpha\theta})\}$$

$$+e^{-\alpha(m-1)T}\{S+(\frac{hD}{\theta}+cD)[\frac{e^{\theta T}}{\alpha+\theta}(1-e^{-(\alpha+\theta)T})$$

$$+\frac{1}{\alpha}(e^{-\alpha T}-1)]\}. \quad \mathbf{(14)}$$

We substitute H/m for T. Then, Eq. (14) can be simplified as follows:

$$TC(m,T_1)$$

$$=\frac{1-e^{-\alpha H(1-\frac{1}{m})}}{1-e^{-\alpha H/m}}\{\frac{hD+cD\theta}{\theta(\alpha+\theta)}e^{\theta T_1}$$

$$+\frac{D\alpha(h+c\theta)+\pi D(\alpha+\theta)}{\alpha^2(\alpha+\theta)}e^{-\alpha T_1}$$

$$+\frac{\pi D}{\alpha}T_1 e^{-\frac{\alpha H}{m}}-\frac{\pi D}{\alpha}e^{-\frac{\alpha H}{m}}(\frac{H}{m}+\frac{1}{\alpha})+(S-\frac{hD+cD\theta}{\alpha\theta})\}$$

$$+e^{-\alpha H(1-\frac{1}{m})}\{\frac{hD+cD\theta}{\theta(\alpha+\theta)}e^{\frac{\theta H}{m}}+\frac{D(h+c\theta)}{\alpha(\alpha+\theta)}e^{-\frac{\alpha H}{m}}$$

$$+(S-\frac{hD+cD\theta}{\alpha\theta})\}. \quad \mathbf{(15)}$$

Let Q_{i-1} be the lot size in ith cycle, where i=1, 2, ..., m. From Eq. (3), the lot sizes for the first cycle and the last cycle are

$$Q_0=I(0)=\frac{D}{\theta}[e^{\theta T_1}-1],$$

$$\text{and } Q_{m-1}=\frac{D}{\theta}[e^{\theta T}-1]+DT_2. \quad \mathbf{(16)}$$

Lot sizes for the remaining cycles, i=2, 3, ..., m−1, are

$$Q_{i-1}=I(0)+DT_2. \quad \mathbf{(17)}$$

III. The Solution Procedure

The purpose of the paper is to determine the optimal values of m and T_1 which minimize $TC(m, T_1)$. For a given value of m, taking the first and second derivative of $TC(m, T_1)$ with respect to T_1, we obtain

$$\frac{dTC}{dT_1}=\frac{1-e^{-\alpha H(1-\frac{1}{m})}}{1-e^{-\frac{\alpha H}{m}}}\{\frac{hD+cD\theta}{\alpha+\theta}e^{\theta T_1}$$

$$-\frac{D\alpha(h+c\theta)+\pi D(\alpha+\theta)}{\alpha(\alpha+\theta)}e^{-\alpha T_1}$$

$$+\frac{\pi D}{\alpha}e^{-\frac{\alpha H}{m}}\}, \quad \mathbf{(18)}$$

and

$$\frac{d^2TC}{dT_1^2}=\frac{1-e^{-\alpha H(1-\frac{1}{m})}}{1-e^{-\frac{\alpha H}{m}}}\{\frac{\theta(hD+cD\theta)}{\alpha+\theta}e^{\theta T_1}$$

$$+\frac{D\alpha(h+c\theta)+\pi D(\alpha+\theta)}{\alpha+\theta}e^{-\alpha T_1}\}>0. \quad \mathbf{(19)}$$

Because $\frac{d^2TC}{dT_1^2}$ is positive, for a given value of m, $TC(m, T_1)$ is convex with respect to T_1, and the first derivative of $TC(m, T_1)$ with respect to T_1 increases as well. Therefore, for a given value of m, the optimal value $T_1^*(m)$ is the unique solution of Eq. (20):

$$\frac{hD+cD\theta}{\alpha+\theta}e^{\theta T_1}-\frac{D\alpha(h+c\theta)+\pi D(\alpha+\theta)}{\alpha(\alpha+\theta)}e^{-\alpha T_1}$$

$$+\frac{\pi D}{\alpha}e^{-\frac{\alpha H}{m}}=0. \quad \mathbf{(20)}$$

Let

$$f(T_1)=\frac{hD+cD\theta}{\alpha+\theta}e^{\theta T_1}-\frac{D\alpha(h+c\theta)+\pi D(\alpha+\theta)}{\alpha(\alpha+\theta)}e^{-\alpha T_1}$$

$$+\frac{\pi D}{\alpha}e^{-\frac{\alpha H}{m}}. \quad \textbf{(21)}$$

Then, f increases with respect to T_1, and $T_1^*(m)$ is the optimal value if and only if $f(T_1^*(m))=0$ for a given value of m. Since $TC(m, T_1)$ is convex with respect to T_1 the Newton-Raphson method can be used to find the optimal value of T_1 for a given value of m. However, it may not be easy for a practitioner with limited mathematical knowledge to understand the Newton-Raphson method. In this section, we shall present a simple algorithm to compute the optimal values of m and T_1. Before describing the algorithm, we need the following theorem.

Intermediate Value Theorem (Thomas and Finney, 1992): Let g be a continuous function on $[u, v]$, and let $g(u)g(v)<0$. Then, there exists a number $d\in(u,v)$ such that $g(d)=0$.

The following algorithm is based on the above theorem and the uniqueness of the root of Eq. (18). The algorithm is commonly known as bisection. Now, we are in a position to outline the algorithm to determine the optimal value $T_1^*(m)$. Note that $f(\frac{H}{m})>0$ and $f(0)<0$. Hence, $0<T_1^*(m)<\frac{H}{m}$ if $m\geq 2$.

The algorithm

Step 1: If $m=1$, set $T_{opt}=H$ and go to Step 7. If $m\geq 2$, go to Step 2.

Step 2: For a given value $m\geq 2$, let $\varepsilon>0$.

Step 3: Set $T_L=0$ and $T_U=\frac{H}{m}$.

Step 4: Let $T_{opt}=\frac{T_L+T_U}{2}$.

Step 5: If $|f(T_{opt})|<\varepsilon$, go the Step 7. Otherwise, go to Step 6.

Step 6: If $f(T_{opt})>0$, set $T_U=T_{opt}$. If $f(T_{opt})<0$, set $T_L=T_{opt}$. Then, go to Step 4.

Step 7: $T_1^*(m)=T_{opt}$ and exit the optimal value.

For a given value of m, the above algorithm can find the optimal value $T_1^*(m)$. The optimal integer m can be selected as follows:

The criterion for selecting the optimal number of replenishments: Let m^* be the smallest integer such that $TC(m^*, T_1^*(m^*))<TC(j, T_1^*(j))$ for all $j=m^*+1$, m^*+2, ..., $m^*+mstop$. Then, we take m^* as the optimal number of replenishments. Consequently, $(m^*, T_1^*(m^*))$ is the optimal solution of $TC(m, T_1)$.

Montgomery (1982) presented a computer program for the optimal economic design of an $\overline{X}$-control chart. Montgomery's search procedure essentially consists of two phases. The second phase of optimization finds the optimal control limit coefficient $\overline{k}$ and the optimal sampling interval $\overline{h}$ for each value of n in the interval $\max\{1, n^*-10\}\leq n\leq n^*+10$, where n^* is the optimal sample size in phase 1 of Montgomery's search procedure. Based on the point of view of Montgomery (1982), we can take $mstop=10$.

IV. A Numerical Example

Let $S=200$, $h=2$, $\pi=10$, $c=20$, $\alpha=0.1$, $\theta=0.08$, $D=1000$, $H=3$ and $\varepsilon=0.001$. The above algorithm is employed to solve Eq. (18) for T_1 to yield the results shown in Table 1. Using the criterion for selecting the optimal number of replenishments, we find that $m^*=9$, $T_1^*(9)=0.2434$, $T^*=0.3333$, $TC(9, T_1^*(9))=3029.0444$. $Q_0=245.7852$, $Q_i=245.7852+89.9=335.6852$ for all $i=1$, 2, ..., 7, and $Q_8=337.7833+89.9=427.6833$.

V. Conclusions

In this paper, we have derived an inventory model for deteriorating items with shortages during a finite planning horizon while considering the time value of money. We have also shown that the present value of the total cost over a finite planning horizon is convex. A simple solution algorithm using a line search method has been presented to determine the optimal interval which has positive inventories. A numerical example has been given to illustrate the solution algorithm.

Table 1. Optimal Solution of the Numerical Example

m	$T_1^*(m)$	T^*	$TC(m, T^*)$
1	3.0000	3.0000	16175.9315
2	1.0672	1.5000	8807.5399
3	0.7195	1.0000	5523.6366
4	0.5427	0.7500	4227.2873
5	0.4355	0.6000	3601.9223
6	0.3637	0.5000	3277.4276
7	0.3123	0.4286	3112.1009
8	0.2735	0.3750	3040.6606
* 9	0.2434	0.3333	3029.0444
10	0.2192	0.3000	3057.8091
11	0.1994	0.2727	3115.0756
12	0.1829	0.2500	3193.1885
13	0.1689	0.2308	3286.9985
14	0.1569	0.2163	3392.9169
15	0.1464	0.2000	3508.3681
16	0.1373	0.1875	3631.4565
17	0.1293	0.1765	3760.7563
18	0.1221	0.1667	3895.1749
19	0.1157	0.1579	4033.8614

References

Aggarwal, V. and H. Bahari-Kashani (1991) Synchronized production policies for deteriorating items in a declining market. *IIE Transactions*, **23**, 185-197.

Bahari-Kashani, H. (1989) Replenishment schedule for deteriorating items with time-proportional demand. *Journal of Operational Research Society,* **40**, 75-81.

Chung, K. H. and Y. H. Kim (1989) Economic analysis of inventory systems: a rejoinder. *The Engineering Economist*, **35**, 75-80.

Chung, K. J. and P. S. Ting (1994) On replenishment schedule for deteriorating items with time-proportional demand. *Production Planning and Control*, **5**, 392-396.

Covert, R. P. and G. C. Philip (1973) An EOQ model for items with Weibull distribution deterioration. *AIIE Transactions*, **5**, 323-326.

Dave, U. (1979) On a discrete-in-time order-level inventory model for deteriorating items. *Journal of Operational Research Society*, **30**, 349-354.

Dave, U. and L. K. Patel (1981) (T, S_i) policy inventory model for deteriorating items with time-proportional demand. *Journal of Operational Research Society*, **32**, 137-142.

Dohi, T., N. Kaio, and S. Osaki (1992) A note on optimal inventory policies taking account of time value. *RAIRO-Operations Research*, **26**, 1-14.

Ghare, P. M. and G. F. Schrader (1963) A model for exponentially decaying inventories. *Journal of Industrial Engineering*, **14**, 238-243.

Goswami, A. and K. S. Chaudhuri (1991) An EOQ model for deteriorating items with shortages and a linear trend in demand. *Journal of Operational Research Society*, **42**, 1105-1110.

Gurnani, C. (1983) Economic analysis of inventory systems. *International Journal of Production Research*, **21**, 261-277.

Hariga, M. A. (1994) Economic analysis of dynamic inventory models with non-stationary costs and demand. *International Journal of Production Economics*, **36**, 255-266.

Hollier, R. H. and K. L. Mak (1983) Inventory replenishment policies for deteriorating items in a declining market. *International Journal of Production Research*, **21**, 813-826.

Montgomery, D. C. (1982) Economic design of an $\overline{X}$-control chart. *Journal of Quality Technology*, **14**, 40-43.

Moon, I. and W. Yun (1993) An economic order quantity model with a random planning horizon. *The Engineering Economist*, **39**, 77-83.

Nahmias, S. (1982) Perishable inventory theory: a review. *Operations Research*, **30**, 680-708.

Raafat, F. (1991) Survey of literature on continuously deteriorating inventory models. *Journal of Operational Research Society*, **42**, 27-37.

Sachan, R. S. (1984) On (T, S_i) policy inventory model for deteriorating items with time proportional demand. *Journal of Operational Research Society*, **35**, 1013-1019.

Shah, Y. K. (1977) An order-level lot-size inventory model for deteriorating items. *AIIE Transactions*, **9**, 108-112.

Thomas, G. B. and R. L. Finney (1992) *Calculas with Analytic Geometry*, 8nd Ed. Addison-Wesley Publishing Company, Inc., Reading, MA, U.S.A.

Ting, P. S. and K. J. Chung (1994) Inventory replenishment policy for deteriorating items with a linear trend in demand considering shortages. *International Journal of Operations and Production Management*, **14**(8), 102-110.

Trippi, R. R. and D. E. Lewin (1974) A present value formulation of the classical EOQ problem. *Decision Sciences*, **5**, 30-35.

在允許缺貨與有限計劃週期之條件下退化性物品存貨系統之經濟分析

鐘崑仁　劉約瑟

台灣工業技術學院工業管理系

摘　要

在允許缺貨、常數需求與固定計劃週期之條件下，本研究探討退化性物品之庫存補充策略。同時這篇論文在採用折扣現金流量之觀點下，提出一線性搜尋法以決定在有庫存量時之最佳週期區間，最後以一個例子來解釋求解過程。